NUMERICAL METHOD FOR CALCULATING SURFACE CURRENT DENSITY ON A THREE-DIMENSIONAL SCATTERER WITH SMOOTH CONTOUR

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1. INTRODUCTION

The Yasuura method, that belongs to the generalized multipole expansion method, has dual algorithms[1]; one is an algorithm for calculating a scattered field and the other is for calculating a total magnetic field on the surface of scatterer from which we can compute a surface current density. Recently, we successfully applied the former to the three-dimensional scattering problem and found some new aspects about this problem[2],[3].

In this paper, we apply the latter to the three-dimensional scattering problem; we present an approximation method for calculating a surface current density on a three-dimensional scatterer. Numerical examples for the bodies of revolution show that the surface current density on the scatterer changes with an observation plane for each scatterer and a shape of scatterer for a fixed observation point.

2. FORMULATION OF THE PROBLEM

Let us consider a three-dimensional perfectly conducting smooth scatterer as shown in Fig.1. The point $P(r,\theta,\phi)$ is in the exterior infinite domain $V$, and the point $Q(r',\theta',\phi')$ is on the surface of the scatterer $S$. The surface current density $J(Q)$, which we seek, equals to the cross product of the unit normal vector $\mathbf{v}(Q)$ and the total magnetic field $H(Q)$;

$$ J(Q) = \mathbf{v}(Q) \times H(Q) $$

We can also get the surface current density using the total tangential magnetic field $H^{t}(Q)$ as

$$ J(Q) = \mathbf{v}(Q) \times H^{t}(Q) $$

where

$$ H^{t}(Q) = H(Q) - (\mathbf{v}(Q) \cdot H(Q)) \mathbf{v}(Q). $$

By the Yasuura method, an approximate total tangential magnetic field on the surface of the scatterer can be represented by a linear combination of the cross product of the unit normal vector on surface and the complex conjugate of the vector spherical wave functions $\mathbf{m}$, $\mathbf{n}$[4] as

$$ H^{t}_n(Q) = \sum_{m=-n}^{n} \left[ c_{mn}(N) \mathbf{v} \times \mathbf{m}^*_m + d_{mn}(N) \mathbf{v} \times \mathbf{n}^*_m \right]. $$

We define the distance between the $H^{t}_n(Q)$ and the $H^{t}(Q)$ in the least squares sense;

$$ \| H^{t}_n - H^{t} \| = \left[ \int_{S} \left| H^{t}_n(Q) - H^{t}(Q) \right|^2 \right]^{1/2}. $$
Since the infinite set of the boundary values of the modal functions is complete in the functional space $L^2(Q)$ consisting of all the square integrable functions on the boundary [5], there exists a sequence $\{H_N^4(Q) : N = 0, 1, 2, \ldots\}$ which converges to $H^4(Q)$ in the mean squares sense:

$$|| H_N^4(Q) - H^4(Q) || \to 0 \quad (N \to \infty).$$

Therefore we choose the coefficients $\{c_{mn}(N), d_{mn}(N)\}$ so as to make Eq.(5) to be minimum. Minimizing the difference of the boundary value and using the reaction formula [6], we have simultaneous linear equations about expansion coefficients. When the scatterer is a body of revolution, for example, we obtain the normal equation as follows;

$$L [c_{p}^{(i)}(N) \alpha_{pqn}^{(i)} + c_{p}^{(i)}(N) \beta_{pqn}^{(i)}] = \gamma_{pq}^{(i)}$$

(7)

where $\alpha_{pqn}^{(i)}$, $\beta_{pqn}^{(i)}$ and $\gamma_{pq}^{(i)}$ (i=1,2,3,4) are the known functions. By using the normal equation or the orthogonal decomposition [7], the coefficients of the modal expansion can be determined. We get the approximate total tangential magnetic field $H_N^4(Q)$ and derive the approximate surface current density $J_N(Q)$ using these coefficients.

3. NUMERICAL RESULTS AND DISCUSSIONS

Numerical computation are curried out for the bodies of revolution whose surface are described by

$$r'(\theta) = a (1 + \delta \cos 3\theta'),$$

(8)

where $a > 0$ and $0 < \delta < 1$ (see Fig.2). In numerical calculation, we divide the interval $[0, \pi]$ into $L$ segments and discretize Eq.(5) with respect to sampling points which are determined according to the Gauss-Legendre quadrature formula by considering the property of the associated Legendre function. To minimize a discretized distance corresponding to Eq.(5) by the QR algorithm reduces to solving simultaneous linear equation about expansion coefficients. Fig.3 shows the approximate surface current density for a fixed $N$ as a function of $L$. From this result, we can make a following choice for $L$ such that $L = 2(N + 1)$. Fig.4 shows the convergence property of the surface current density $J_N(Q)$ as $N$ increases, although the more a deformation parameter $\delta$ increases, the more a convergence rate becomes slow. This figure show the validity of this method.

As a numerical example, the amplitudes and the phases of surface current density, when a plane wave is incident on the scatterer from a rotational axis of the objects, are shown in Fig.5. The precision of data is better than three significant figures for each observation point. First we compare a surface current density on the observation plane $\phi_o = 0$ with $\phi_o = \pi / 2$. On $\phi_o = 0$ plane, the surface current density has $r$ and $\theta$ components, that is, the direction of
surface current density is tangential direction in $\phi = 0$ plane. On the other hand, in the case of $\phi_o = \pi / 2$ plane, the surface current density has only $\phi$ component. The amplitudes of surface current density make big differences between $\phi_o = 0$ plane and $\phi_o = \pi / 2$ plane and the phases at $\phi_o = 0$ plane are similar to one at $\phi_o = \pi / 2$ plane as shown in Fig.5. Next we pay attention to a dependence of the surface current density on shapes of the scatterer. This dependence is clearly observed in the amplitudes of surface current density at the illuminated side of the scatterer but is not at the shadowed side both $\phi_o = 0$ plane and $\phi_o = \pi / 2$ plane.

4. CONCLUSIONS

We present an approximation method for a surface current density on the three-dimensional scatterer and confirm from the numerical results that the Yasuura method for calculating a surface current density is effective even for the three-dimensional scattering problem.

References

Fig. 3 The surface current density as a function of $L/(N+1)$.

(a) $N=9$  (b) $N=14$  ($k\alpha=3, \delta=0.05, \varphi_i=\theta_i=\alpha=0, \phi_o=0, \theta_o=\pi/6$)

Fig. 4 The surface current density versus inverse of the number of truncation. ($k\alpha=5, \varphi_i=\theta_i=\alpha=0, \phi_o=\theta_o=0$)

Fig. 5 The surface current density on the bodies of revolution.

(a) $\phi_o=0$ plane  (b) $\phi_o=\pi/2$ plane  ($k\alpha=5, \varphi_i=\theta_i=\alpha=0$)

--- $\delta=0.00$, --- $\delta=0.05$, --- $\delta=0.10$, --- $\delta=0.15$