abstract. The principle to measure the ionospheric height and propagation distance of atmospherics, with very high accuracy, is represented based on the dispersion characteristics of the Earth-ionosphere waveguide. It is shown by computer simulation that the measurement error for the cut-off frequency is less than 1% and that for propagation distance is less than 6% even if the duration of the signal data with strong noise, whose amplitude ratio of noise to signal is 0.5, is extremely short such as 13 msec.

1. Introduction.
Tweek atmospherics are VLF/ELF electromagnetic waves which are observed very frequently during nighttime, and the propagation characteristics of tweek atmospherics are found to be explained in terms of the Earth-ionosphere wave propagation theory (Outsu, 1960; Budden, 1961; Wait, 1970). It has been expected that the propagation characteristics of tweek atmospherics can be utilized to yield the information on the lower ionosphere such as the ionospheric height, density profile and plasma dynamics of the lower ionosphere. Nevertheless, there have been developed no effective analysis methods for the dispersion effect of tweeks (Outsu, 1960) and also there have been errors in locating the sources of atmospherics (e.g., Iwai et al., 1979).

However, atmospherics generated by lightening events are very useful waves to investigate the propagation properties of waveguide between the Earth and lower ionosphere (D-region) because its region very effective reflecting layer for VLF radio waves (Volland et al., 1987; Reeve and Rycroft, 1972), though it seems that the usage of tweek atmospherics is unsuccessful in the detailed study of the lower ionosphere.

The present paper proposes a new method to determine, with sufficiently high accuracy, the ionospheric average height over the propagation path and its distance on the basis of the dispersion properties of the Earth-ionosphere waveguide propagation of atmospherics, and we discuss the effectiveness of our method by computer simulation.

2. Signal of atmospherics.
Atmospherics are VLF/ELF electromagnetic waves propagated in the Earth-ionosphere waveguide (Budden, 1961; Wait, 1970), after being transmitted from lightening discharges. Both of the ionosphere and ground are approximately perfect conductor for the frequency range below a few kilo Hertz (Volland, 1984), and so the condition of these reflecting walls is not effective for the charge of phase of sferics without the attenuation (Shimakura et al., 1992). Then the group velocity $v_g$ of the 1st order mode of the atmospherics propagating in the Earth-ionosphere waveguide is given by,

$$v_g \simeq c \sqrt{1 - \left( \frac{\pi}{k_0 h} \right)^2}$$

where $k_0 = \frac{\omega}{c}$ is the propagation constant of electromagnetic wave in free space, $h$ the
ionospheric height and c the light velocity. Then the propagation time of the atmospherics at a angular frequency \( \omega \) is given by the following relation:

\[
t - t_0 = \frac{d}{v_g} = \frac{d}{c} \frac{1}{\sqrt{1 - \left(\frac{\pi}{k_0 h}\right)^2}}
\]

(2)

where \( t_0 \) is the generation time of lightening discharge and \( d \) the propagation distance of atmospherics. And then the instantaneous frequency \( f(t) \) is obtained by using eq.(2),

\[
\begin{align*}
    f(t) &= \frac{f_c(t - t_0)}{\sqrt{(t - t_0)^2 - \left(\frac{\hat{d}}{c}\right)^2}} \\
    f_c &= \frac{f_c}{2h}
\end{align*}
\]

(3)

where \( f_c \) is the cut-off frequency of the 1st order mode of the Earth-ionosphere waveguide. Integrating eq.(3) leads to the phase \( \phi(t) \) of the signal of atmospherics at the time \( t \).

\[
\phi(t) = \int 2\pi f(t) dt = 2\pi f_c \sqrt{(t - t_0)^2 - \left(\frac{\hat{d}}{c}\right)^2} + \text{const.}
\]

(4)

Namely, the signal \( x(t) \) of atmospherics (1st order mode) propagated in the Earth-ionosphere waveguide is given by the following equation, with taking into account the temporally slowly varying amplitude \( a(t) \) and phase \( \phi_0(t) \),

\[
x(t) = a(t) \cos \left\{ 2\pi f_c \sqrt{(t - t_0)^2 - \left(\frac{\hat{d}}{c}\right)^2} + \phi_0(t) \right\}
\]

(5)

The first term of the phase in eq.(5) gives the component with fast frequency variation, while the slowly varying variation, \( a(t) \) and \( \phi_0(t) \), correspond to the spectrum broadening.

3. Principle to estimate the ionospheric height and propagation distance of atmospherics with high accuracy.

Atmospherics are non-stationary signals with very short duration (a few tens to a few hundreds of milliseconds) and very fast frequency variation. And signal component near the cut-off frequency is unfortunately very weak because of the strong attenuation in the Earth-ionosphere waveguide, though the ionospheric height can be estimated on the basis of the cut-off frequency, \( f_c \) of the waveguide from the relation of eq.(3). So, it is very difficult to estimate the ionospheric height by means of ordinary spectrum analysis for tweek atmospherics.

As shown in eq.(5), the dispersion characteristics of atmospherics are given in terms of the cut-off frequency \( f_c \), propagation distance \( d \) and the generation time of lightening discharge \( t_0 \). Then the pseudo-signal of atmospheric \( \hat{x}(t) \) with dispersion parameters, \( f_c \), \( d \) and \( t_0 \) may be expressed by,

\[
\begin{align*}
    \hat{x}_c(t) &= \cos \{ \hat{\phi}(t) \} \\
    \hat{x}_s(t) &= \sin \{ \hat{\phi}(t) \} \\
    \hat{\phi}(t) &= 2\pi f_c \sqrt{(t - t_0)^2 - \left(\frac{\hat{d}}{c}\right)^2} \left( 2\pi - 502 \right)
\end{align*}
\]

(6)
Considering eqs. (5) and (6), the product of the observed signal, \( x(t) \), and the pseudo-signal, \( \hat{x}(t) \) or \( \hat{x}'(t) \), can be given as the sum of two signal components. One is a signal component with the slowly varying phase and the other a component with fast frequency variation about twice that of the observed signal. Only the signals, \( y_c(t) \) and \( y_s(t) \), with slowly varying phase are utilized for the parameter estimation by means of the low pass filter. Signals \( y_c(t) \) and \( y_s(t) \) are given by the following equations:

\[
\begin{align*}
y_c(t) &= \frac{a(t)}{2} \cos\{\phi(t) - \hat{\phi}(t)\} \\
y_s(t) &= -\frac{a(t)}{2} \sin\{\phi(t) - \hat{\phi}(t)\}
\end{align*}
\] (7)

From eq.(7), the phase of observed signal, \( \phi(t) \), can be estimated by

\[
\phi(t) = \hat{\phi}(t) + \tan^{-1}\left\{\frac{-y_s(t)}{y_c(t)}\right\}
\] (8)

It is shown by eq.(8) that the phase of non-stationary atmospheric signal can be measured by using pseudo-signal, with very high accuracy.

Now, let determine the error function \( \varepsilon \) \((f_c, d, t_0, K)\), including additional parameter \( K \), as follows.

\[
\begin{align*}
\varepsilon &= \sum_k \left\{(\phi_k + K)^2 - \hat{\phi}_k^2\right\}^2 \\
\phi_k &= \hat{\phi}(t_k) \\
\hat{\phi}_k &= 2\pi f_c \sqrt{\left(t_k - t_0\right)^2 - \left(\frac{d}{c}\right)^2}
\end{align*}
\] (9)

In eq.(9), \( \phi_k \) is the phase at the time, \( t_k \), to be measured by eq.(8), and parameters, \( f_c, d, t_0 \) and \( K \), are those to be estimated. Under the condition, \( \phi_k + K - \hat{\phi}_k \geq 0 \), the solutions by which the error \( \varepsilon \) has minimum value can be easily obtained by solving the following simultaneous linear equations.

\[
\begin{pmatrix}
2\sum_k \hat{\phi}_k & -4\pi^2 \sum_k t_k^2 & 8\pi^2 \sum_k t_k & N \\
2\sum_k \hat{\phi}_k t_k & -4\pi^2 \sum_k t_k^3 & 8\pi^2 \sum_k t_k^2 & \sum_k t_k \\
2\sum_k \hat{\phi}_k t_k^2 & -4\pi^2 \sum_k t_k^4 & 8\pi^2 \sum_k t_k^3 & \sum_k t_k^2 \\
2\sum_k \hat{\phi}_k t_k^3 & -4\pi^2 \sum_k t_k^5 & 8\pi^2 \sum_k t_k^4 & \sum_k t_k^3 \\
\end{pmatrix}
\begin{pmatrix}
K \\
f_c \\
f_c^2 t_0 \\
\end{pmatrix}
= \begin{pmatrix}
K^2 \\
K^2 + 4\pi^2 f_c^2 \left(\frac{d}{c}\right)^2 - t_0^2 \\
\end{pmatrix}
\]

\[
\begin{pmatrix}
-\sum_k \hat{\phi}_k^2 \\
-\sum_k \hat{\phi}_k t_k \\
-\sum_k \hat{\phi}_k t_k^2 \\
-\sum_k \hat{\phi}_k t_k^3 \\
\end{pmatrix}
\] (10)
4. Computer simulation
Atmospheric signal data including noise is constructed in terms of the equation \( x(t) + \alpha z(t) \), where \( x(t) \) is the atmospheric signal with the amplitude of unity given by eq. (5), \( z(t) \) the random noise with the value distributed uniformly in the range from 1 to -1, and \( \alpha \) the noise factor which gives the intensity of noise. And the atmospheric signal generated by the Earth-ionosphere waveguide with the cut-off frequency \( f_c = 1.7 \text{kHz} \) and propagation distance \( d = 6000 \text{km} \). When the data to be analyzed has sufficiently long duration and do not include noise, the maximum measurement errors of parameters are negligible, which is less than 0.0027 % for the cut-off frequency, and less than 0.041 % for the propagation distance. And, though the error increases with enhancement of noise, the measurement accuracy for the parameters is sufficiently high when the duration of signal data is long enough. Figure 1 shows the measurement error versus noise factor \( \alpha \) for the data with extremely short duration of 13 msec. Even for this case the measurement errors of the parameters are sufficiently small.

Fig. 1 The measurement errors of the dispersion parameters versus noise factor \( \alpha \) for the simulated signal data with extremely short duration of 13 msec.

references