PROCEEDINGS OF ISAP '96, CHIBA, JAPAN

COUPLED-MODE ANALYSIS OF COUPLED MULTICONDUCTOR MICROSTRIP LINES

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1. Introduction

Multiconductor transmission lines arranged in a multilayered dielectric medium are widely used in the design of microwave and millimeter-wave integrated circuits. One of the important subjects on such transmission systems is to evaluate efficiently as well as accurately the high-frequency electromagnetic coupling between nearby conductor lines. The coupling causes a crosstalk that affects seriously the circuit performance in high speed operation. The transmission characteristics of coupled conductor lines can be rigorously analyzed using various numerical methods[1]. However those direct solution methods become much involved both analytically and numerically when the number of conductor lines increses. In this respect, we have recently proposed a coupled-mode theory[2] for multilayered and multiconductor transmission lines based on the full-wave analysis. In this approach, the total fields supported by multiconductor lines are approximated by a linear combination of the modal fields associated with the isolated single conductor lines, and the coupled-mode equations governing the evolution of amplitudes of currents on each line are systematically derived by making use of the reciprocity relation[3].

The purpose of this paper is to apply the proposed coupled-mode theory to the analysis of various coupled microstrip structures and to confirm the validity of the approximation. The dispersion characteristics of two nonidentical coupled microstrip lines and N identical coupled microstrip lines are calculated using the coupled-mode theory. It is shown that the results are in very close agreement with those of the rigorous Galerkin's moment method solutions over a broad range of weak to strong coupling, indicating that the coupled-mode theory yields a good approximation with enough accuracy.

2. Formulation

We consider first two coupled nonidentical microstrip lines as shown in Fig. 1. Two microstrips a and b of widths $2w_a$ and $2w_b$ and zero thickness are situated with spacing $2d$ on the substrate-cover interface in a trilayered structure, which consists of a ground plane of perfect conductor, a dielectric substrate of thickness h and relative permittivity ε_r, and a cover layer of free space. Let β_ν, $\tilde{e}_\nu(\zeta;y)$ and $\tilde{h}_\nu(\zeta;y)$, and $\tilde{j}_\nu(\zeta)$ ($\nu = a, b$) be the the propagation constant, the eigenmode fields, and the eigenmode current in the Fourier transformed domain for the fundamental EH_0 mode of the isolated single microstrip ν. Then we have the following coupled-mode equations [2]:

$$\frac{d}{dz}\begin{pmatrix} a \\ b \end{pmatrix} = -i \begin{pmatrix} 1 & \frac{N_{ab}+N_{ba}}{2} \\ \frac{N_{ab}+N_{ba}}{2} & 1 \end{pmatrix}^{-1} \begin{pmatrix} \beta_a & \beta_a \frac{N_{ab}+N_{ab}}{2} + K_{ab} \\ \beta_b \frac{N_{ab}+N_{ba}}{2} + K_{ba} & \beta_b \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \tag{1}$$

with

$$N_{\nu\mu} = -\frac{2}{\pi}\int_0^\infty I_{\nu\mu}(\zeta)\cos(2\zeta d)d\zeta, \quad K_{\nu\mu} = \frac{i}{4\pi}\int_0^\infty L_{\nu\mu}(\zeta)\cos(2\zeta d)d\zeta \tag{2}$$

$$I_{\nu\mu}(\zeta) = \int_0^\infty [\tilde{e}_{\nu,x}(\zeta;y)\tilde{h}_{\mu,y}(\zeta;y) + \tilde{e}_{\nu,y}(\zeta;y)\tilde{h}_{\mu,x}(\zeta;y)]dy \tag{3}$$

$$L_{\nu\mu} = \tilde{e}_{\nu,x}(\zeta,h)\tilde{j}_{\mu,x}(\zeta) + \tilde{e}_{\nu,z}(\zeta,h)\tilde{j}_{\mu,z}(\zeta) \quad (\nu,\mu = a,b) \tag{4}$$

where $a(z)$ and $b(z)$ are the amplitude functions of currents on the microstrips a and b, and the eigenmode fields have been normalized so that $N_{aa} = N_{bb} = 1$.

Next we consider the coupled-mode equations for multiple identical coupled microstrip lines. Fig. 2. shows the cross section of three identical coupled microstrips a, b, and c of width $2w$ and zero thickness, which are situated with equal spacing $2d$ on the substrate-cover interface in a trilayered structure. Let β_0, $\tilde{e}(\zeta,y)$ and $\tilde{h}(\zeta,y)$, and $\tilde{j}(\zeta)$ be the the propagation constant, the eigenmode fields, and the eigenmode current in the Fourier transformed domain for the fundamental EH_0 mode of the single microstrip in isolation. Then we obtain the following coupled-mode equation [2]:

$$\frac{d}{dz}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = -i \begin{pmatrix} 1 & N_1 & N_2 \\ N_1 & 1 & N_1 \\ N_2 & N_1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} \beta_0 & \beta_0 N_1 + K_1 & \beta_0 N_2 + K_2 \\ \beta_0 N_1 + K_1 & \beta_0 & \beta_0 N_1 + K_1 \\ \beta_0 N_2 + K_2 & \beta_0 N_1 + K_1 & \beta_0 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} \tag{5}$$

with

$$N_1 = -\frac{2}{\pi}\int_0^\infty P(\zeta)\cos(2\zeta d)d\zeta, \quad N_2 = -\frac{2}{\pi}\int_0^\infty P(\zeta)\cos(4\zeta d)d\zeta \tag{6}$$

$$K_1 = \frac{i}{4\pi}\int_0^\infty Q(\zeta)\cos(2\zeta d)d\zeta, \quad K_2 = \frac{i}{4\pi}\int_0^\infty Q(\zeta)\cos(4\zeta d)d\zeta \tag{7}$$

where

$$P(\zeta) = \int_0^\infty [\tilde{e}_x(\zeta,y)\tilde{h}_y(\zeta,y) + \tilde{e}_y(\zeta,y)\tilde{h}_x(\zeta,y)]dy \tag{8}$$

$$Q(\zeta) = \tilde{e}_x(\zeta,h)\tilde{j}_x(\zeta) + \tilde{e}_z(\zeta,h)\tilde{j}_z(\zeta). \tag{9}$$

where $a(z)$, $b(z)$, and $c(z)$ are the amplitude functions of currents on the microstrips a, b, and c. Equations (1) and (5) show that the problem of coupled microstrip lines is reduced to the eigenvalue problems for the coupling coefficients matrix, which can be easily solved using a standard computation program for matrix equations. The eigenvalues give the propagation constants of coupled modes in the coupled microstrip lines, and the associated eigenvectors determine the excitation ratios of currents on the individual lines for the respective coupled-modes.

3. Numerical results

The coupling coefficients $K_{\nu\mu}$ and $N_{\nu\mu}$ $(\nu,\mu = a,b)$ for two nonidentical coupled microstrip lines and K_1, K_2, N_1, and N_2 for three identical coupled microstrip lines are given by the simple overlap integrals between the eigenmode fields and currents in Fourier transformed domain for the individual isolated single microstrips. The eigenmode fields and currents of the isolated single microstrip can be easily calculated using Galerkin's moment method in the spectral domain. The integrals in Eqs.(3) and (8) can be evaluated in closed form using the dyadic Green's function in the spectral domain. The integrals in Eqs. (2) , (6), and (7) are efficiently calculated using the spectral data which were obtained in Galerkin's moment method analysis of the conventional single microstrip line. The normalized propagation constants β/k_0 of the even and odd EH_0 modes of two nonidentical coupled microstrip lines calculated by Eq. (1) are given in TABLE 1 for $w_a = 1.5mm, w_b = 2.0mm, h = 0.635mm, \varepsilon_r = 9.8, f = 10$ and $20GHz$, and various separations d/w_{av} $(w_{av} = (w_a + w_b)/2)$, and compared with those of the direct Galerkin's moment method solutions, where k_0 is the wavenumber in free space. Similarly the normalized propagation constants β/k_0 of the three fundamental EH_0 modes calculated by the coupled-mode equations (5) are given in TABLE 2 for $w = 1.5mm, h = 0.635mm, \varepsilon_r = 9.8, f = 10$ and $20GHz$, and various separations d/w. These numerical results were obtained by expanding the transverse and longitudinal current components in terms of the lowest six Chebyshev polynomials weighted by appropriate edge factors. We can see that the results for both configurations show a very close agreement with those of the rigorous Galerkin's moment method solutions over a broad range of the separation. Eqs. (6) and (7) reveal that the coupling effects are described only by the integrand factors $\cos(2\zeta d)$ and $\cos(4\zeta d)$, which depend on the separation distance between two microstrips concerned. Therefore the coupled-mode equations for N identical coupled-microstrip lines with equal spacing can be easily deduced by the analogy to Eqs. (5)-(9). Fig. 3. shows dispersion curves of the fundamental N modes calculated using the coupled-mode theory for N identical coupled-microstrip lines with $w = 1.5mm, h = 0.635mm, \varepsilon_r = 9.8$, and $d/w = 1.3$. It is worth emphasizing that even for $N = 10$, the computation time of the coupled-mode analysis is almost same as in the numerical analysis by Galerkin's moment method for a conventional single microstrip.

4. Conclusion

We have presented a coupled-mode theory for the coupled microstrip lines in trilayered structure on the full-wave analysis. The coupled-mode equations for two nonidentical coupled microstrip lines and three identical coupled microstrip lines have been obtained in closed form. The coupling coefficients are given by the simple overlap integrals between the eigenmode fields and currents in the spectral domain for the individual isolated single lines. This greatly simplifies the computational procedure and therefore remarkably reduces the computation time. The numerical results of the propagation constants are in very close agreement with those of the rigorous Galerkin's moment method solutions over a broad range of weak to strong coupling. This fact suggests that the coupled-mode theory is an efficient analytical and numerical technique to characterize the high-frequency crosstalk in highly integrated microwave circuits.

Acknowledgment This work was supported in part by the Grand-in-Aid for Scientific Research (B) from the Ministry of Education, Science and Culture, Japan.

References

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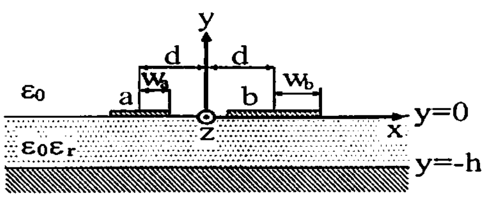


Fig. 1 Cross section of two nonidentical coupled microstrip lines.

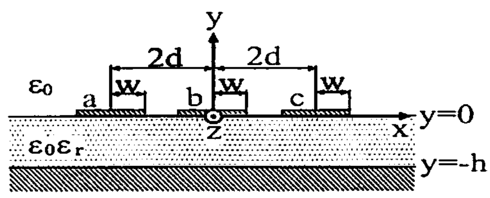
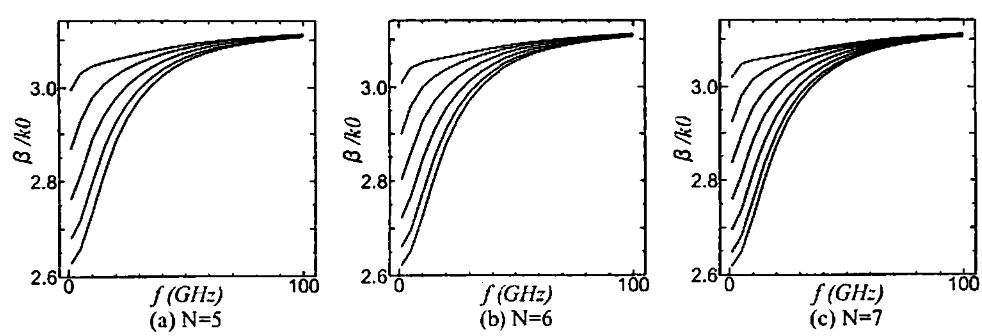
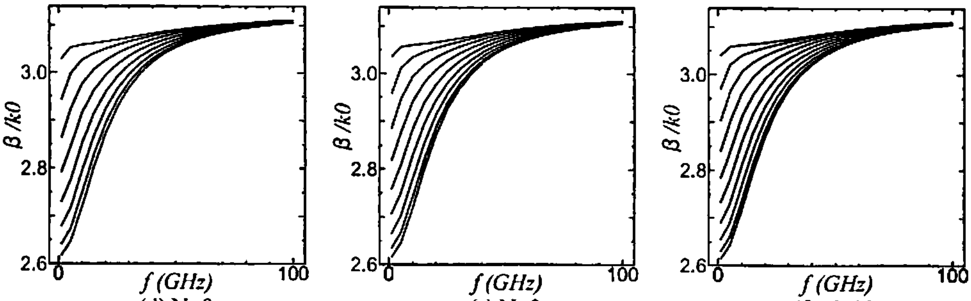


Fig. 2 Cross section of three identical coupled microstrip lines.





(d) N=8 (e) N=9 (f) N=10

Fig. 3 Dispersion curves of the fundamental N modes calculated using the coupled mode theory for N identical coupled microstrip lines with $w = 1.5mm, h = 0.635mm, \varepsilon_r = 9.8$, and $d/w = 1.3$.

TABLE 1 Normalized propagation constants β/k_0 of the even and odd EH_0 modes of two nonidentical coupled microstrip lines with $w_a = 1.5mm, w_b = 2.0mm, h = 0.635mm, \varepsilon_r = 9.8$. CMT and MOM refer to the present coupled-mode theory and the direct Galerkin's moment method.

$(a) 10GHz(\beta_a/k_0 = 2.89439, \beta_b/k_0 = 2.94206)$

	Even Mode				Odd Mode			
d/w_{av}	1.10	1.30	1.50	2.00	1.10	1.30	1.50	2.00
CMT	3.00037	2.99366	2.97199	2.94553	2.73983	2.83933	2.87087	2.89186
MOM	3.01726	2.99168	2.96730	2.94487	2.77494	2.83692	2.86722	2.89139

$(b) 20GHz(\beta_a/k_0 = 2.97776, \beta_b/k_0 = 3.01480)$

	Even Mode				Odd Mode			
d/w_{av}	1.10	1.30	1.50	2.00	1.10	1.30	1.50	2.00
CMT	3.05215	3.03734	3.02232	3.01465	2.89359	2.95411	2.97217	2.97774
MOM	3.06466	3.03771	3.02110	3.01473	2.92050	2.95696	2.97214	2.97784

TABLE 2 Normalized propagation constants β/k_0 of the three fundamental EH_0 modes of three identical coupled microstrip lines with $w = 1.5mm, h = 0.635mm, \varepsilon_r = 9.8$.

	$(a) 10GHz(\beta_0/k_0 = 2.89439)$				$(b) 20GHz(\beta_0/k_0 = 2.97776)$			
	1st Symmetric Mode				1st Symmetric Mode			
d/w	1.10	1.30	1.50	2.00	1.10	1.30	1.50	2.00
CMT	3.05905	3.01694	2.97562	2.92196	3.08526	3.04486	3.01351	2.98383
MOM	3.04599	3.01297	2.97473	2.92194	3.08215	3.04637	3.01438	2.98387
	1st Asymmetric Mode				1st Asymmetric Mode			
d/w	1.10	1.30	1.50	2.00	1.10	1.30	1.50	2.00
CMT	2.88127	2.88861	2.89176	2.89399	2.97568	2.97728	2.97765	2.97776
MOM	2.88103	2.88851	2.89172	2.89399	2.97558	2.97726	2.97765	2.97776
	2nd Symmetric Mode				2nd Symmetric Mode			
d/w	1.10	1.30	1.50	2.00	1.10	1.30	1.50	2.00
CMT	2.69520	2.76157	2.80895	2.86607	2.83599	2.90071	2.93910	2.97157
MOM	2.66121	2.75540	2.80767	2.86603	2.83059	2.90328	2.94017	2.97161