I. Introduction

There are several criteria for antenna evaluation. Particularly, the authors have studied on the efficiency-bandwidth product (EB) for small dielectric loaded antennas (DLAs) [1]-[3]. In general, the EB takes almost the same value among antennas having the same electrical volume. The performance of the antenna is therefore considered to be enhanced when its EB has been increased keeping the same electrical volume as before. To evaluate the EB, its dependence on the permittivity of the DLAs is of particular interest. In [2], the existence of the peak value in the Wheeler’s radiation power factor (PF, i.e. the EB) [1] was analytically predicted when the permittivity of the dielectric loading was changed. Although the DLA in [2] was an infinitesimal electric dipole located at the center of a dielectric sphere, other shapes for the dielectric loading are also of interest to investigate the permittivity dependence of the EB.

The EB is simple in its expression and only needs a radiation resistance and an input reactance of an antenna. This feature of the EB is very advantageous because one can measure the radiation resistance of the small antennas with a sufficient precision by using the Wheeler cap method [4]-[7]. The experimental considerations on the EB have, however, rarely been reported.

In this paper, the EBs of small cylindrical dielectric loaded monopole antennas (DLMAs) are measured and dependence of the EB on the permittivity of the dielectric loading is investigated. For the measurement, small radiation resistances of the DLMAs are precisely measured on the basis of the Wheeler cap method. Finally, permittivity dependence of the EB is discussed from the viewpoints of the measured results as well as a mathematical reasoning.

II. Measurement of the EB

The EB is expressed as the following equation

\[
EB = \frac{R_r}{R_r + R_{loss}} \cdot \frac{1}{X} = \frac{R_r}{X} \cdot \frac{1}{R_r + R_{loss}}.
\]

(1)

where \( R_r \), \( R_{loss} \) and \( X \) are the radiation resistance, conductor loss and input reactance of an antenna, respectively. From (1), one needs only a ratio of the radiation resistance to the input reactance of the antenna in order to obtain the EB. This simple expression is useful for both experiment and calculation.

![Fig. 1 Dielectric loaded monopole antenna (DLMA)](image_url)
Input impedances of the DLMAs shown in Fig.1 were measured at 2GHz ($\lambda_0=150\text{mm}$, $\lambda_0$ is a wavelength in vacuo). Measurement of the radiation resistance was done with an HP8510C network analyzer. However, accurate measurement of a small resistance combined with a large reactance is very difficult. The input resistances are very sensitive to various errors such as random error, systematic error and, especially, drift error because the input resistances are obtained from the reflection coefficients. In particular, its fluctuation causes unacceptable errors to the small resistances under 1$\Omega$, where the reflection coefficient is nearly unity. The main cause of this time variation appears to be the drift error of the instrument. Although the random noise can be considerably removed by averaging multiple measurements, the drift error is difficult to remove because the error is a function of time and is caused by thermal characteristics of the instrument. One way to overcome this difficulty is to cancel the effect of the drift by measuring some quantity twice and evaluating the difference between them, like measurement of the radiation resistance in the Wheeler cap method.

The Wheeler cap method is capable of extracting the correct value of the radiation resistance from the time varying small resistances. The process of the Wheeler cap method is to measure the input resistances with and without the cap. Then the radiation resistance is given by subtracting the input resistance with the cap from the one without cap. The major advantage of the method is cancellation of both the systematic error [7] and the drift error. Furthermore in the present study, we used a stub tuner to measure the input resistances for improvement of measurement precision. Figure 2 shows the setup geometry. The size of the cap was 400 x 500 x 200mm$^3$. Interconnecting cables were made to be as short as possible. A semi-rigid cable of 360mm long was used to keep the measurement system stable. The procedure of the measurement is as follows. (i) The stub tuner is connected between the antenna and the network analyzer. (ii) The input impedance is tuned to nearly 50$\Omega$ without the cap. (iii) The antenna is shielded with the cap and the input resistance is measured. (iv) After disconnecting the antenna from the stub tuner, S-parameter of the stub tuner is measured. (v) The input impedance of the antenna is thereby calculated from the matched impedance and the S-parameter of the stub tuner.

On the other hand, the input reactance of the DLMA was very stable. This is because the reflection coefficient was nearly unity, hence radial fluctuation of the reflection coefficient due to the drift error gave only small error to the input reactance. The input reactances with and without the cap were measured and averaged.

Fig. 2  The setup for measurement of input impedance

III. Results

Figures 3 and 4 show permittivity dependence of the radiation resistances $R_r$ and the inverse of the input reactance $1/X$, respectively. In addition to the measured results, the calculated results by the FDTD method are presented. Here, the conductor loss of the Wheeler cap is approximated to be negligible. We obtained a good agreement between the measurement and the calculation. Figure 5 shows the measured EBs. The values of the EBs are normalized to those of the bare monopole. Each error bar corresponds to a standard deviation obtained from the 10-15 samples of measurements. According to Figures 3-5, sufficient precision was achieved to estimate the EB dependence on the permittivity of the dielectric loading. An enhancement of performance compared with the bare monopole antenna in terms of the EB value was obtained in this range of permittivity. To note, there
seems to be an optimal value of permittivity to obtain the best performance in this range. Because the volume, or physical dimension, of all the DLMAs is the same, Fig. 5 also means that electrically larger antenna is not always necessary to obtain the better value of the EB. The reason is explained in the next section.

**Fig. 3** Radiation resistances $R_r$ of the DLMAs  

**Fig. 4** Inverse of the input reactance $1/X$ of the DLMAs

**Fig. 5** EBs of the DLMAs (----: EB of the bare monopole)

**IV. Discussion**

The enhancement of the EB over the bare dipole as well as the existence of the peak value are similar phenomena in the dielectric loaded dipole antenna of [2]. In this study, the maximum improvement of the measured EB is about 25% over the bare monopole at the relative permittivity $\varepsilon_r \approx 5$. Meanwhile in [2], the maximum improvement of the EB is about 11% at $\varepsilon_r = 2.0$. The reason for the existence of the peak value in the EB is as follows.

Assume that the permittivity dependence of the radiation resistance is similar to the case of the spherical dielectric loaded antenna in [2], as the permittivity of the loading becomes higher. Then deducing from [2], the radiation resistance as a function of $\varepsilon_r$ may be approximated as

$$R_r(\varepsilon_r) = \frac{a}{(\varepsilon_r + b)^2}$$

where the constants $a, b$ are real and positive. Meanwhile, the capacitance in the DLMA is proportional to the permittivity of the dielectric loading. The term $1/X = \omega C$ ($\omega$ angular frequency, $C$: capacitance in the antenna) thus increases approximately linearly with an increase in the $\varepsilon_r$ of the dielectric loading. Then, a function $f = 1/X$ may be approximated as
\[ f(\varepsilon_r) = \frac{1}{X} = c \varepsilon_r + d \]  
\text{(3)}

where the constants \( c, d \) are real and positive.

Consequently, a product of the radiation resistance and \( 1/X \) becomes a derivable function of the \( \varepsilon_r \). Hence, the maximum point of the EB exists at a certain \( \varepsilon_r \). The value of \( \varepsilon_{r(\text{max})} \), which gives the maximum EB, is

\[ \varepsilon_{r(\text{max})} = b - \frac{2d}{c}. \]  
\text{(4)}

Note that the peak of the EB curve in Fig. 5 only appears when \( \varepsilon_{r(\text{max})} > 1 \). Equation (4) indicates that the \( \varepsilon_{r(\text{max})} \) depends on the steepness of a decrease in the \( R_r \). In other words, a steeper decrease in the \( R_r \) or a larger value of the \( b \) in (2) gives a higher value of the \( \varepsilon_{r(\text{max})} \). Such characteristics of the \( \varepsilon_{r(\text{max})} \) may be explained in a comparison between the spherical dielectric loaded dipole in [2] and the present DLMAs. In [2], the dipole is loaded with a dielectric sphere, hence the dielectric loading in the direction of the maximum radiation is thicker than the present cylindrical case if the volume of the dielectric loading is made to be the same each other. The radiation resistance is therefore slower to degrade than in [2] as the \( \varepsilon_r \) of the loading becomes high. Consequently, the peak of the EB curve shifts to the higher \( \varepsilon_r \) according to (4). As a result, in such DLMAs, the dielectric loading of high permittivity is not always essential to obtain a better EB, because the radiation resistances of the DLMAs continue to decrease as the permittivity of the loading becomes high.

V. Conclusions
The EBs of the DLMAs have experimentally and numerically been shown to improve over the bare monopole antenna when they are loaded with relatively low permittivity of the cylindrical dielectric. The small radiation resistances have precisely been measured using the Wheeler cap method. Finally, we have explained the reason of the EB improvement as well as the existence of the peak value with lower permittivity of the dielectric loading. The dependence of the EB on the shape of the dielectric loading is, however, left for a future study.

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References