1. Introduction

This paper describes results of joint research between the International Research Centre for Telecommunications-transmission and Radar (IRCTR) at the Delft University of Technology (The Netherlands) and the Kiev International University of Civil Aviation (Ukraine).

A set of mathematical models is developed to describe features of the rain microstructure and turbulence within the shower as can be measured with Doppler polarimetric radar. Computer simulation of the Doppler-polarimetric spectra computed from the coherent polarimetric radar signal is done on the basis of these models. The generalized model uses the rain microstructure and turbulence parameters as initial data. The model gives Doppler spectra at different combinations of linear polarization of the transmitted and received waves. In particular, the differential reflectivity $Z_{dr}(v)$ and linear depolarization ratio $L_{dr}(v)$ are calculated as functions of Doppler velocity $v$. The model establishes the relationships between the microstructure of and turbulence in rain and the Doppler-polarization parameters of the radar signal. Verification of the model is done using the Delft Atmospheric Research Radar (DARR), which is able to measure $Z_{dr}(v)$ and $L_{dr}(v)$ as radar observables. The results obtained show qualitatively and quantitatively the relationships between measured Doppler-polarimetric parameters and turbulence intensity using both physical and statistical considerations.

2. Model

The approach for modeling and simulation has been developed in our joint research and assumes no preliminary suppositions about statistical models of the received signals. The Gamma dropsize distribution is used as initial model of rain microstructure, the shape of the droplet is considered as function of equivalent drop diameter, and the Kolmogorov-Obukhov (so called minus five third) law describing the spatial spectrum of turbulence is applied to derive the dynamic behavior of scatterers in the radar resolution volume.

In case the scatterers are completely affected by the motion of air, the Doppler spectrum of the reflected signal could be related with turbulence parameters being one of the main reasons of the motion of scatterers. However, the supposition that droplets are perfectly involved into turbulent motion is a kind of idealization. On the one hand, the larger the drop diameter the greater is the contribution of the drop to the reflected signal power. But on the other hand, the larger the drop diameter the less the dynamics of this drop are related with just the turbulent motion because of the inertia of drop. That is why motions of large drops can inadequately contribute to turbulent motions of air. Probably, this is a most difficult problem in case the turbulence intensity should be retrieved from radar signal. The worked out model solves the problem of drop inertia. As an intermediate result, the drop fall velocity distribution, drop turbulence velocity distribution, and distribution of drop velocity caused by both turbulence and gravity are derived and calculated using the software developed.

The model provides Doppler spectra $S_{mn}(v)$ at different combinations of polarization for transmit (first index) and receive (second index) with $m=x;y$, $n=x;y$, $x$ and $y$ denote the linear orthogonal polarization basis. In the special case of horizontal-vertical polarization basis, $x$ equals h (horizontal) and $y$ equals v (vertical). The model then provides three Doppler-polarimetric spectra: $S_{hh}(v)$, $S_{hv}(v)$, $S_{vh}(v)$. Polarization observables are described by the following expressions:
$Zdr(v) = 10 \log \left[ \frac{S_{hh}(v)}{S_{vv}(v)} \right], \quad Ldr(v) = 10 \log \left[ \frac{S_{hv}(v)}{S_{vv}(v)} \right]$ (1)

Initial information for simulation includes models and parameters of drop size distribution, other characteristics of rain microstructure and atmospheric turbulence, as well as parameters of the radar as wavelength, range resolution, antenna beam width, elevation angle.

Thus, three kinds of modeled Doppler spectra are obtained for analysis. The first one is calculated without turbulence and is caused by gravity only; the second one is caused by turbulence of a given intensity but without taking into account the particle fall velocities; the third one is caused by both turbulence and gravity. So we can calculate the dependencies of the co-polar and cross-polar signal at different polarizations without turbulence or with turbulence of known intensity in order to research the relationships between turbulence and the Doppler-polarimetric parameters.

The model is implemented as software written in language C++ using the Borland C Builder program medium. The software is suitable for Windows 95/98 and works as a multithreaded application with a comfortable menu system.

3. Measurements

The measured data are obtained with Delft Atmospheric Research Radar DARR, which is an S-band frequency-modulated continuous wave Doppler-polarimetric weather radar [1] located on the top of the 23-floor building.

The rain event with low rain intensity (less than 1 mm/h) was used for comparison. The data are characterized by the following parameters:

- Maximum range is 9525 m (effectively top of the precipitating cloud is at about 6000 m)
- Radar range resolution is about 90 m
- Maximum Doppler velocity is ± 6 m/s, Doppler resolution is 4.7 cm/s
- Doppler polarimetric measurement time is 0.96 s (it relates to three complex spectrum images for three types of polarization $S_{hh}$, $S_{hv}$, and $S_{vv}$ versus Doppler and range)
- Radar antenna elevation $\theta$ is 30 degrees.
- Number of images of each range cell that can be averaged is 20

Processing software allows to select Doppler spectrum data in any range cell for one of the 20 images or to average Doppler spectra on any number of images from 2 up to 20. Further analysis is done afterwards. In case of averaging the power images $<S_{hh}S_{hh}>$, $<S_{hv}S_{hv}>$, $<S_{vv}S_{vv}>$ versus range and Doppler velocities are available. Polarization parameters $Zdr$ and $Ldr$ are determined in dB as

$$Zdr=10\log \left( \frac{<S_{hh}S_{hh}>}{<S_{vv}S_{vv}>} \right), \quad Ldr=10\log \left( \frac{<S_{hv}S_{hv}>}{<S_{vv}S_{vv}>} \right)$$ (2)

An other kind of processing is done without averaging but using a smoothing algorithm applied on a single image. The processing of $hh$, $vv$ and $hv$ spectra includes selecting the data that exceed the predefined threshold which depends on the signal-to-noise ratio.

4. Comparison of modeled and measured data

The measured Doppler velocity consists of the projection of the velocities of hydrometeors on the line-of-sight and of the wind Doppler velocity. The analyzed DARR data are slant profile measured with an azimuth direction orthogonal to the mean horizontal wind direction. But this horizontal wind direction is quite changing with height and is unknown. That is why a superposition of measured and modeled data is necessary in order to compare them and take the mentioned horizontal wind phenomena into consideration. The mean Doppler velocity of raindrops predicted by the model was used to shift the measured Doppler spectrum of the averaged reflectivity. Figure 1 represents direct comparison of the experimental ($E$) differential reflectivity $EZdr$ and the modeled differential reflectivity $Zdr$ versus Doppler velocity.

It is visible that experimental data are characterized with quite a large variability but a clear trend versus Doppler velocity is seen, and a rather good coincidence of this trend with modeled curve can be noted. Variations of model parameters allow finding the modeled curve that fit to the data actually in every case.
In order to make a quantitative analysis, both modeled and measured results are approximated by linear regression, and then the parameters of the approximated line (slope and intercept) are analyzed. The behavior of $Z_{dr}$-slope versus Doppler spectrum width at horizontal polarization for modeled and measured data has been studied. Experimental results contain some points with $\Delta Vh < 1$ m/s, while all modeling results correspond to $\Delta Vh > 1$ m/s. This can be explained by the singularity of data processing when only "above threshold" parts of measured spectra were processed, and as a result of this the estimates of the spectrum width determined on the threshold level 0.5 could be understated. But in general, a similarity of the two dependencies can be found.

5. Relationship between turbulence intensity and Doppler polarimetric parameters

The relationship between eddy dissipation rate $Eps \text{ [cm}^2\text{/s}^3\text{]}$ and Doppler spectrum width $\Delta Vh \text{ [m/s]}$ is shown in Figure 2. The correlation coefficient equals 0.978. The relationship between $slope_{Zdr}$ and eddy dissipation rate $Eps \text{ [cm}^2\text{/s}^3\text{]}$ is presented in Figure 3. The correlation is here -0.96.

The close relation between the slope of $Zdr(v)$ and Doppler spectrum width is illustrated in Figure 4. Slope and intercept of $Ldr(v)$ linear regression are also related with eddy dissipation rate. Correlation coefficients for them equal -0.813 and 0.821 correspondingly. However the absolute values of $Ldr$ are very small, and the function $Ldr(v)$ is difficult to be measured with good accuracy.
The model developed provides a direct technique for obtaining the relationship between Doppler spectrum width and eddy dissipation rate, while DARR measurements do not allow such possibility. Nevertheless, to implement some comparison, we assume that the measured Doppler spectrum width at \textit{hh} polarization $E \Delta V_h$ is tightly connected with turbulence intensity (see Figure 2) and that the Doppler spectrum width at \textit{vv} polarization $E \Delta V_v$ is just an informative parameter to be measured. The relationship $E \Delta V_v$ [m/s] versus $E \Delta V_h$ [m/s] is designated in Figure 5 by boxes.

The dotted line represents the relationship between Doppler spectrum width $\Delta V_h$ [m/s] and logarithm of eddy dissipation rate $\varepsilon$ [cm$^2$/s$^3$], which is designated by $x = \log(\varepsilon)$ along the horizontal axis. It is interesting to note that the two relationships are similar when the Doppler spectrum width is more than 1 m/s and that the two relationships almost coincide if a small shift in $x$ of 0.5 is included.

### 6. Discussion

Determination of the relationships between the radar echo-signal and parameters of clouds and precipitation is one of the most difficult problems of atmospheric remote sensing. The important practical outcomes can be obtained by the combining experiments with mathematical modeling and simulation. In particular, the developed model allows selecting some Doppler-polarimetric parameters to retrieve turbulence parameters from the radar signal. Both physical and statistical considerations of obtained results show that when we take the Doppler spectrum width and the slope of the differential reflectivity as Doppler-polarimetric parameters (which can be used for the retrieval of turbulence parameters in precipitation) they can be used to measure the eddy dissipation rate. The developed approach to the Doppler-polarimetric modeling is original, has a good perspective for further development and can play an important role in atmospheric radar remote sensing data interpretation and signal processing. The developed models, approaches and algorithms should be applied to new weather radars, first of all to the Transportable Atmospheric Radar TARA, which is under development in IRCTR.

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### References