1. Introduction

In micro-cellular mobile communications, diffraction by flat-topped lossy obstacles is quite important in field strength prediction when the buildings have cubical shapes. In most cases, the building surfaces form right angle lossy wedges which are of great interests. In many literature, e.g. [1] and [2], the Luebbers’ heuristic diffraction coefficient for a lossy wedge [3] has been applied. In [4], another new heuristic diffraction coefficient has been proposed. However, detailed comparisons for right angle lossy wedges in [5] show that both the Luebbers’ and Holm’s diffraction coefficients have very large deviations compared with the results derived by using rigorous Maliuzhinets diffraction coefficient and the Finite Difference Time Domain (FDTD) method [6], especially in the illumination regions. The large difference has also been shown in [7] and [8] with the comparisons of the diffraction field derived by the Luebbers’ and rigorous Maliuzhinets’ diffraction coefficients. For a flat-topped perfectly conducting (PEC) obstacle, the uniform theory of diffraction (UTD) and Fresnel-Kirchhoff theory [9] and [10] have been applied to solve the diffraction field, and the later can only be used when small wave length and paraxial approximations are satisfied [11]. For a flat-topped lossy obstacle, the Luebbers’ heuristic slope diffraction coefficient was derived in [12] based on Luebbers’ heuristic diffraction coefficient which has been proven to be not accurate as foregoing mentioned. In the present paper, a simplified Maliuzhinets’ diffraction coefficient for a right angle wedge is considered to give the rigorous diffraction field over a flat-topped lossy obstacle.

2. Maliuzhinets’ Diffraction Coefficient For Right Angle Lossy Wedges

The Maliuzhinets’ diffraction coefficient is available in [7] for far field case and it can be further simplified using the work reported in [4], [13] and [14] to get an easily calculated from given by

\[
D = \frac{\psi(n\pi/2 + \pi - \phi)}{\psi(n\pi/2 - \phi')} [D^{(1)} - D^{(3)}] + \frac{\psi(n\pi/2 - \pi - \phi)}{\psi(n\pi/2 - \phi')} [D^{(2)} - D^{(4)}]
\]  

(1)

where \(\phi'\) and \(\phi\) are the incident and diffracted angles, respectively (Fig. 1), \((2-n)\pi\) is the interior angle of the wedge and \(D^{(l)}\) \((l = 1, \cdots, 4)\) is expressed as [4]

\[
D^{(l)} = D'(L,n;\phi,\phi') = -\frac{e^{-j\pi/4}}{2n\sqrt{2\pi k}} \cot \gamma^{(l)} F_0 \left(2kLn^2 \sin^2 \gamma^{(l)} \right)
\]  

(2)

where \(k\) is wavenumber and \(F_0\) is the transition function defined in [15]. The expressions for \(\gamma^{(l)}\) \((l = 1, \cdots, 4)\) are available in [4], \(L\) is distance parameter equal to \(s\) or \(s'/s\) for plane or spherical wave incidence, respectively, where \(s\) and \(s'\) are shown in Fig. 1. In (1), \(\psi(\alpha)\) is the Auxiliary function expressed as [16]

\[
\psi(\alpha) = \psi_{n\pi/2}(\alpha + n\pi/2 + \pi/2 - \theta_0) \cdot \psi_{n\pi/2}(\alpha - n\pi/2 - \pi/2 + \theta_0) - \psi_{n\pi/2}(\alpha + n\pi/2 - \pi/2 + \theta_0) \cdot \psi_{n\pi/2}(\alpha - n\pi/2 + \pi/2 - \theta_0)
\]  

(3)

For a right angle lossy wedge \(\psi_{n\pi/2}(\alpha)\) is available in [16] and it can be further simplified as

\[
\psi_{n\pi/2}(\alpha) = 1 + 2\cos(\alpha/3)/(3\cos(\alpha/6))
\]  

(4)
where $\varepsilon_r$ and $\sigma$ are the relative permittivity and conductivity of the lossy wedge. $\varepsilon_0$ is the permittivity of free space. In (3), $\theta_{0,n}$ can be derived using $\sin \theta_{0,n} = \sqrt{\varepsilon_r - j \sigma / \omega \varepsilon_0}$ and $\sin \theta_{0,n} = 1/\sqrt{\varepsilon_r - j \sigma / \omega \varepsilon_0}$ for TM and TE waves [4][16], respectively, if $\varepsilon_r$ is not close to 1. This condition can be satisfied because $\varepsilon_r$ is greater than 3 for the materials at microwave frequencies in urban and indoor micro-cellular communications [1], e.g. for dry brick and dry concrete, $\varepsilon_r$ is within 4–6. We should mention that (1) is valid for the far-field case, hence it is applicable to most environments in micro-cellular mobile communications at microwave frequencies.

3. Diffraction by Flat-Topped Lossy Obstacles

As seen in Fig. 2 and assume that spherical wave incidence, the higher order diffraction field by the two right angle wedges is [4]

$$E_d^m = E_0 e^{-jkx_0} \frac{s_T}{s_T} \sum_{s_1,s_2,s_3} \frac{1}{s_1s_2s_3} \left[ \frac{1}{jks_{s}} \right]^m \partial^m D_1 \partial^m D_2$$

(5)

where $E_0$ is the relative amplitude of a spherical source. $D_1$ and $D_2$ are the diffraction coefficient for the right angle wedges 1 and 2, respectively. $s_T = s_1 + s_2 + s_3$ and $m = 0, 1, \ldots$. For slope diffraction ($m = 1$) field

$$E_d^{slope} = E_0 e^{-jkx_0} \frac{s_T}{s_T} \left( D_1D_2 - \frac{1}{jks_{s}} \frac{\partial D_1}{\partial \phi_1} \frac{\partial D_2}{\partial \phi_2} \right)$$

(6)

The Maliuzhinets’ slope diffraction coefficient is given by

$$d = \frac{1}{jk} \frac{\partial D}{\partial \phi'} = \frac{1}{jk} \left[ \frac{\psi(n\pi/2 + \pi - \phi) \partial}{\psi(n\pi/2 + \phi') \partial D^1 \partial D^2} \left[ D^{(1)} - D^{(3)} \right] + \frac{\psi(n\pi/2 - \pi - \phi) \partial}{\psi(n\pi/2 - \phi') \partial D^1 \partial D^2} \left[ D^{(2)} - D^{(4)} \right] \right]$$

(7)

where $D$ is the diffraction coefficient for an arbitrary lossy wedge. $\partial D^{(l)}/\partial \phi' = \partial D^{(l)}/\partial \gamma^{(l)}$. $\partial \gamma^{(l)}/\partial \phi'$, and considering $F_0'(x) = j[F(x) - 1] + F(x)/(2x)$ when $\partial D^{(l)}/\partial \gamma^{(l)}$ is calculated. $\partial \gamma^{(l)}/\partial \phi'$ $(l = 1, \ldots, 4)$ is easy to be derived from the expression of $\gamma^{(l)}$ $(l = 1, \ldots, 4)$ available in [4]. In (7), $\psi'(x) = d\psi/dx$, and in (6), $\partial D/\partial \phi$ is calculated by using

$$\frac{\partial D}{\partial \phi} = jkd - \frac{\psi(n\pi/2 + \pi - \phi) \left[ D^{(1)} - D^{(3)} \right]}{\psi(n\pi/2 + \phi') \left[ D^{(2)} - D^{(4)} \right]} \left[ \frac{\psi'(n\pi/2 - \pi - \phi)}{\psi'(n\pi/2 - \phi')} \right] \cdot D$$

(8)

where $\partial D^{(l)}/\partial \phi = \partial D^{(l)}/\partial \gamma^{(l)}/\partial \phi$ and $\partial \gamma^{(l)}/\partial \phi$ $(l = 1, \ldots, 4)$ is also easy to get from [4].

4. Numerical Results

Figs. 3 and 4 show the diffraction field considering the first ($m = 0$) and second order ($m = 1$) diffraction field for flat-topped lossy obstacles at 5 GHz frequency band under plane and spherical wave incidence, respectively. Let us assume that $E_0 = 1$ for simplicity. It is seen that the total diffraction field is not continuous. This is due to that we just consider the second order diffraction field which is quite lower in the shadowing region. In this case, the higher order diffraction field should be considered to remain the total diffraction field continuous.

5. Conclusion

The diffraction field by flat-topped lossy obstacles is solved by using rigorous Maliuzhinets’ slope diffraction coefficient derived in the present paper. It is easy to be programmed and has very fast
calculation speed. However, as a future work, higher order diffraction field should be considered to remain the diffraction field continuous at deep shadowing region

Acknowledgement. This work was funded by the Academy of Finland.

References

Fig. 1 Geometry of a lossy wedge diffraction

Fig. 2 Geometry of the diffraction by a flat-topped lossy obstacle

Fig. 3 Diffraction field by flat-topped lossy obstacles at 5 GHz under plane wave incidence. The relative permittivity is $\varepsilon_r = 8$ and the conductivity is $\sigma = 0.001 \text{ S/m}$. $s_2 = s_3 = 100\lambda$. (a) $\phi_1 = 10^0$ as shown in Fig. 2. (b) $\phi_1 = 120^0$.

Fig. 4 Diffraction field by flat-topped lossy obstacles at 5 GHz under spherical wave incidence $\varepsilon_r$ and $\sigma$ are as in Fig. 3, and $s_1 = s_2 = s_3 = 100\lambda$. (a) $\phi_1 = 10^0$. (b) $\phi_1 = 120^0$.