1. Introduction

A ray tracing technique is frequently utilized for electromagnetic wave propagation modeling. Ray tracing or ray launching algorithms are very powerful providing excellent outputs. On the other hand the calculation is usually very complex and time consuming. One of the main disadvantages, especially for commercial applications, is a necessity of precise geometrical inputs, e.g. building database for urban scenarios. When an inhomogeneous environment is considered, both the algorithm and the geometrical inputs tend to be even more complicated.

In this paper a multi-agent approach to combinatorial optimization problems - the Ant Colony Optimization - is introduced for a simple ray tracing performed on only an ordinary bitmap describing a two-dimensional scenario. This bitmap can be obtained as a simple scan where different colors represent different mediums. Using the presented algorithm the fastest path for a ray (electromagnetic wave) can be found according to the Fermat's principle. In this way the wave propagation modeling could become, for selected tasks, easy and fast to apply. Two examples of applications of the proposed algorithm are described as well.

2. Ant Colony Optimization

The Ant Colony Optimization proposed by Dorigo [1] is a multi-agent approach for various combinatorial optimization problems (see [2] for many references). The algorithms were inspired by the observation of real ant colonies. Ants are social insects living in colonies with interesting foraging behavior. In particular, an ant can find shortest paths between food sources and a nest. While walking from food sources to the nest and vice versa (Fig. 1a), ants deposit on the ground a substance called pheromone, forming a pheromone trail (Fig. 1b). Ants can smell pheromone and, when choosing their way, they tend to choose paths marked by strong pheromone concentrations. It has been shown that this pheromone trail following behavior can give rise to the emergence of shortest paths.

![Fig. 1. Ant Colony Optimization: ants searching the shortest route using a pheromone trail](image)

As it is shown in Fig. 1, when an obstacle breaks the ants’ path, ants try to get around the obstacle randomly choosing either way. If the two paths encircling the obstacle have different lengths, more ants on their continuous pendulum motion pass the shorter route in the same time interval. While each ant keeps marking its way by pheromones the shorter route attracts more pheromone concentrations and consequently more and more ants
choose this route. This feedback leads soon to final stage, where entire ant colony uses the shortest path.

3. Description of the algorithm

The algorithm is based on the Ant Colony System (ACS) technique [5] originally proposed for Traveling Salesman Problem. It was modified for needs of the wave propagation simulation. The adapted algorithm is described in five following steps:

**Step 1 - an undirected graph generation**

An undirected graph $G = (N, A)$, where $N$ is set of nodes and $A$ the set of arcs connecting the nodes, and two nodes $N_1$ and $N_2$ to be connected by the required fastest ray have to be defined in the first step. The nodes $N$ are easily generated as a uniform grid applied on an input pixel bitmap describing the propagation environment. The density of the nodes determines the precision but also memory and computation time demands of the algorithm. Then arcs $A$ interconnect all the nodes of the graph - one with each other. Using simple raster graphics procedures all the nodes and arcs interfering with colored elements, e.g. black, which represent buildings or other obstacles, are eliminated. In addition, each arc must traverse through only a single medium (only one color in the bitmap). Appropriate wave traveling time is assigned to each arc based on its physical length and the medium. The graph generation is shown in Fig. 2ab (in Fig. 2a a very sparse grid of nodes was used as an illustration). All arcs of the graph $G$ are initialized with a small amount of pheromone $\tau_0$, which is a direct wave traveling time between $N_1$ and $N_2$ considering the wave speed in vacuum.

![Graph Generation](image)

**Step 2 - a colony of ants is launched**

In the second step $C$ ants are sequentially launched from $N_1$. $C = 50$ was used for the number of ants in the colony. Each ant walks pseudo-randomly from a node to node via connecting arcs (Fig. 2c) as far as the $N_2$ or dead end is reached. When deciding which arc to go from a specific node, each $i$-th arc leading from the node is assigned a probability:

$$ p_i = \frac{\tau_i \eta_i^\beta}{\sum_i \tau_i \eta_i^\beta} $$

where $\tau_i$ is the pheromone concentration on the $i$-th arc, $\eta_i$ is an a priori available heuristic value for the $i$-th arc, which is a wave traveling time to appropriate connected node and then continuing directly to $N_2$ considering the wave speed in vacuum, and $\beta$ is a parameter determining the relative influence of the heuristic information. This parameter plays a key role for the algorithm performance. The best results were achieved for $\beta = 3.0$. Then a random number $q$ between 0 and 1 is generated. If $q < q_0$, where $q_0 = 0.5$ is another parameter of the algorithm, the ant chooses to go via an arc with the highest probability $p_i$. Otherwise random selection of the arc based on the probability distribution (1) is accomplished. The previously
visited arcs are excluded. After having crossed the selected \(i\)-th arc during the ant’s tour construction a local update rule is immediately applied to the pheromone concentration on the arc:

\[
\tau_i = (1 - \rho) \tau_i + \rho \tau_0 ,
\]

where \(\rho\) is a parameter \(0 \leq \rho \leq 1\). The effect of the local updating rule is to make already chosen arc less desirable for a following ant. In this way more route variations can be explored. \(\rho = 0.2\) was found as an optimal value.

**Step 3 - deterministic optimization of ants’ paths and the best ant selection**

When all ants from the colony finish their routes, the most successful ant (the ant with the fastest path from \(N_1\) to \(N_2\)) is selected to update the pheromone trails in the following step 4. Before the selection a deterministic optimization is performed on all of the ants’ paths. This very fast and simple procedure tries to eliminate unnecessary nodes on an ant’s route. It is based on the following principle: When \(N_a, N_b\) and \(N_c\) are consecutive nodes, the existence of a direct connection between \(N_a\) and \(N_c\) is tested. If such an arc exists, the \(N_b\) is skipped. Overall performance of the algorithm is significantly improved using this technique.

**Step 4 - a global update of pheromone trails**

Pheromones on all arcs of the graph are updated using a global update rule:

\[
\tau_i = (1 - \alpha) \tau_i + \alpha \tau_T ,
\]

where \(\alpha\) is a parameter \((0 \leq \alpha \leq 1)\) determining the evaporation of pheromone concentrations and \(\tau_T\) is the best ant’s traveling time in case of an arc visited by the best ant or zero for all other arcs. Similarly to parameter \(\rho\) the value \(\alpha = 0.2\) was found as an optimum.

In homogenous environments, where the wave speed is the same throughout the map, a value inversely proportional to the path length of the best solution can be used for the pheromone update. In this case the shortest path instead of the fastest path is searched. It leads to the same results in homogenous mediums.

There are two different ways to choose the best ant that is allowed to perform the global updating. In the “global-best” method only the ant that did the fastest route since the very beginning of the optimization process is selected. In the “iteration-best” method always the best ant from the colony of \(C\) ants deposits the pheromones despite of previous iterations. In all simulations presented in this paper iteration-best strategy was applied.

**Step 5 - the algorithm termination**

Steps 2 to 4 are repeated for a fixed number of iterations or as long as the desired solution is reached. After termination of the algorithm the stored solution of the very best ant indicates the fastest path between \(N_1\) and \(N_2\).

4. Evaluation of the algorithm

The algorithm was tested on several simple scenarios. Using these simulations optimal values of the parameters from (1), (2) and (3) were established. The values were already given above. The parameters are slightly different from [5], where the original ACS algorithm is used for Traveling Salesman Problem \((C = 10, q_0 = 0.9, \beta = 2.0, \rho = 0.1, \text{ and } \alpha = 0.1)\). Two basic applications of the algorithm are presented as an illustration:

**Simple ray tracing in inhomogeneous environment**

Fig. 3a shows a simple scenario with three different mediums with a different wave propagation velocity \(v_1 > v_2 > v_3\) (as it is marked in the picture). The mediums are differentiated by colors in the input bitmap of the optimization algorithm. A solid line represents a result of the simulation: the fastest path of the wave from the lower-left corner to the upper-right corner of the bitmap (the path that minimizes the wave traveling time). The Fermat’s principle can be nicely demonstrated. The global solution was usually obtained within 50 iterations for the graph with more than 71,000 arcs.
Path-loss calculations in microcells using the recursive model

Using the presented ACS algorithm together with the Berg’s recursive model [4] a non-line-of-sight path loss could be calculated without any need of building database.

The recursive model is a semi-deterministic approach for street microcells. The model does not require knowledge of building materials but only a two-dimensional building database is needed. The shortest path along streets is determined among buildings between a base station and a mobile antenna. A simple geometrical ray tracing technique can be used. The path is break down into a number of straight segments interconnected by nodes. An “illusory” distance is obtained recursively as function of a real length of segments and an angle of segments crossings in nodes. It means each time the path bends the illusory distance is lengthened in comparison to the physical distance. Then a very simple empirical formula is applied on the illusory distance to calculate the total path loss.

Classical ray tracing would require some kind of building database, at least a location and shape of the buildings. When the ACS algorithm is applied to find the desired shortest segmented path along streets, only a bitmap (scan of the city map) can be used as an input. In this way the coverage predictions for urban microcells could become extremely easy and fast to apply. Fig. 3b demonstrates the shortest path search among several obstacles.

5. Summary

For a simple two-dimensional ray tracing in inhomogeneous mediums a new algorithm based on the Ant Colony Optimization was introduced. A common bitmap can be used as an input describing even complicated environments. Two applications of the technique were presented. First simulations proved a very promising efficiency and usefulness of the algorithm. The work continues tuning and evaluating the method on more complex tasks.

Acknowledgements

This work has been supported by the Grant Agency of the Czech Republic grant No. 102/01/D117 and by the research program No. 212300014.

References