Improved Closed-Form Green’s Function Method for Microstrip Structures

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1. Introduction

Recently, in order to accelerate the calculation of spatial domain Green’s functions for microstrip structures, various closed-form Green’s function methods[1~5] were proposed for application to the method of moments(MoM). Among them there were a new class of closed-form Green’s functions[4,5] which have a more convenient form than the conventional one[1~3]. One key advantage of the functions[4,5] is simplicity in mathematical form that leads to closed-form MoM matrix elements which can be analytically evaluated even without any procedures of the numerical evaluation. For convenience, the functions will be called G-E(Ge-Esselle) closed-form Green’s functions hereafter to distinguish from the conventional functions[1~3]. However, in [4], somewhat burdensome problems such as difficulty in determining adequate approximation parameters and the advance investigation of spectral Green’s functions associated with the variation of the interested frequency range have been still observed. In the present study, an improved approach based upon a three level approximation and the steepest descent path(SDP) method for the Sommerfeld integral(SI) is considered to alleviate such difficulties.

2. Difficulties in the Previous Method

In this section, G-E closed-form Green’s function method [4] is briefly reviewed and the difficulties in the method are discussed. In general, the Green’s functions of planar microstrip structures can be expressed in the following representative forms:

\[
G(\rho) = \int_{-\infty}^{\infty} H_0^{(2)}(k_\rho \rho)k_\rho \tilde{G}(k_\rho) dk_\rho, \quad (1a)
\]

\[
G(\rho) = \int_{-\infty}^{\infty} \tilde{g}(k_\rho) e^{-jk_\rho \rho} dk_\rho, \quad (1b)
\]

where \( H_0^{(2)} \) is the Hankel function of the second kind and order zero. The function \( \tilde{G}(k_\rho) \) is the spectral-domain Green’s functions, which can be obtained analytically. These integrals, called the SI, cannot be evaluated analytically. In [4], \( G(k_\rho) \) is approximated in terms of the sum of exponentials along the two approximation paths \( L_1^{a} \) and \( L_1^{b} \), as shown in Figure 1, and then (1) are analytically transformed by using the formula for semi-infinite integrals of Bessel functions as follows:

\[
G(\rho) \approx \sum_{i=1}^{M} \frac{c_i d_i}{(d_i^2 + \rho^2)^{3/2}} + \sum_{j=1}^{M} \frac{a_j b_j}{(b_j^2 + \rho^2)^{3/2}} \quad (2)
\]

where \( c_i \) and \( a_j \) (\( d_i \) and \( b_j \)) are the complex coefficients(exponents) obtained from the procedure of approximation technique, and the real parts of the complex exponents \( d_i \) and \( b_j \) should be positive.
Fig. 1. The Sommerfeld integration path $L_0$ along the real axis (dotted line), and deformed paths $L_1^1$ and $L_2^1$ (solid line) used in [4].

However, some difficulties have been observed as follows:

1) The improper choice of the approximation parameters, which should be determined in approximating $G(k_\rho)$ along the paths $L_1^1$ and $L_2^1$, yields the complex exponents $d_i$ and $b_j$ with the negative real part, and so troublesome approximation procedures should be repeated until the proper parameters for the expression (2), where all the real parts of $d_i$ and $b_j$ are positive, can be chosen.

2) Even though the proper parameters are chosen after investigating $G(k_\rho)$ in advance, Green’s functions of (2) may give somewhat inaccurate evaluation results. The cause of these difficulties is thought to be due to the choice of the approximation paths that are decided by considering only the accuracy of approximation for $G(k_\rho)$ on the $k_\rho$-plane. Therefore it is necessary to reconsider the approximation paths shown as $L_1^1$ and $L_2^1$ in Figure 1 based upon the theoretical considerations such as robust approach proposed in [3] and the SDP method of SI in [6].

3. The Present Method

Based upon the robust approach in [3] for the derivation of the conventional closed-form Green’s function, the present approximation paths which are composed of the paths $L_1^1$, $L_2^1$ and $L_3^1$ are proposed, as shown in Figure 2. That is, in the present study, the path $L_3^1$ is newly introduced, in order to provide the compensation for the contribution beyond $t_1$ as mentioned in [3], which has not been taken into account in the previous method[4]. So, in this method that can be called three level approach, the parametric equations of the three approximation paths are chosen to be

\begin{align}
L_1^1 : k_\rho &= k_0 (t + jt_0) \quad 0 \leq t \leq t_0 \label{eq:3a} \\
L_2^1 : k_\rho &= k_0 \left[ t + \frac{(t_1 - t)t_0}{t_0(t_1 - t_0)} \right] \quad t_0 \leq t \leq t_1 \label{eq:3b} \\
L_3^1 : k_\rho &= k_0 \left( t + t_0 \right) \quad t_1 \leq t \leq t_2 \label{eq:3c}
\end{align}

where $T_0$ corresponds to the gradient of the path $L_1^1$ and $t_0$ is the value on the real axis of complex $k_\rho$-plane corresponding to the end(truncation) point of the path $L_1^1$, while $t_1$ and $t_2$ are the truncation point of the path $L_2^1$ and $L_3^1$ respectively.

Fig. 2. The paths $L_1^1$, $L_2^1$ and $L_3^1$ used in the present method.
And then we are to discuss how we select the (approximation) parameters $t_0$, $t_1$, $t_2$ and $T_0$, because the adequate choice of those has a very important influence in obtaining the good results.

A. The optimum choice of $T_0$

The optimum choice of $T_0$ which corresponds to the gradient of the path $L_1^1$ is thought to be related to the SDP for the efficient evaluation of SI[6], because it is seen that there is the saddle point at $k_ρ = 0$ in case of the SI expressed as the representative form of (1b). Following the method in [6], it can be easily found that the path $L_1^0$ in case that $\alpha = \pi / 4$, where $\alpha$ is the radial angle as indicated in Figure 2, corresponds to the SDP. Therefore, $T_0 = 1$ (i.e., $\alpha = \pi / 4$) is chosen as the optimum value in the present method.

B. The proper choice of the rest parameters

In order to illustrate how the parameters $t_0$, $t_1$ and $t_2$ are chosen, the present method is applied to the example of the open microstrip structure $(\varepsilon_r = 12.6)$ in [1]. Here, $\varepsilon_r = 12.6$ (GaAs) is chosen to correspond to the dielectric constant widely used in multilayered planar structures for monolithic millimeter and microwave integrated circuits (MMIC). For the example of geometry in [1], the main procedure for the choice of the parameters is summarized as follows:

1) Choose $t_2$ such that $t_2 = 2000$, in order to ensure that the quasi-static contribution of $\hat{G}(k_ρ)$ for very large $k_ρ$ is captured.

2) Choose $t_0$ and $t_1$ such that $t_0 < 1$ and $t_1 > \sqrt{\varepsilon_r}$. This is because it has been observed that the other choices of $t_0$ such that $1 < t_0 < \sqrt{\varepsilon_r}$ yield frequently the complex exponents with the negative real part in approximating $\hat{G}(k_ρ)$ along the path $L_1^0$ as well as the path $L_1^1$. The reason for this is thought to be due to the presence of the surface wave poles (SWP) involved in the path $L_1^0$ and $L_1^1$. After investigating only once the spectral Green’s functions at the center of the interested frequency range, the proper $t_0$ and $t_1$ can be easily determined. Here $t_0 = 0.5$ and $t_1 = 20$ are chosen, and these choices are not so critical. Then $\hat{G}(k_ρ)$ is approximated in the three-level through the similar procedure as described in [4].

As a result, the present closed-form Green’s functions are derived to be

$$G(\rho) = \sum_{i=1}^{N_1} \frac{2a_{i1}b_{i1}}{(b_{i1}^2 + \rho^2)^{3/2}} + \sum_{i=1}^{N_2} \frac{2a_{i2}b_{i2}}{(b_{i2}^2 + \rho^2)^{3/2}} + \sum_{i=1}^{N_3} \frac{2a_{i3}b_{i3}}{(b_{i3}^2 + \rho^2)^{3/2}},$$

where $a_{i1}$, $a_{i2}$ and $a_{i3}$, $b_{i1}$, $b_{i2}$ and $b_{i3}$ are the complex coefficients (exponents) obtained from the procedure of approximation, $N$ is the number of complex exponentials and subscript 1, 2 and 3 mean results obtained through the path $L_1^1$, $L_1^2$ and $L_1^3$ respectively.

4. Numerical Results and Conclusion

In order to check the validity and accuracy of numerical evaluation results of the Green’s function, the present method has been applied to the example of geometry in [1] over the wide frequency based upon the parameters mentioned in the foregoing section. In Figure 3, for the case of three different frequencies (5GHz, 20GHz, 40GHz), numerical evaluation results for the magnitude of the vector and scalar potential Green’s functions obtained by the present method (dotted lines) are compared with the exact results obtained by the numerical integration (solid lines). Excellent agreements between them (i.e., the difference is generally less than 0.5%) are observed over the wide frequency range and spatial range. It should be noted that even a choice of one set of approximation parameters (i.e., $T_0 = 1$, $t_0 = 0.5$, $t_1 = 20$, $t_2 = 2000$), facilitated more accurate results over the wide frequency range. Moreover, in spite of the variation of interested frequency range, the present method didn’t yield the complex exponents (i.e., $b_{i1}$ and $b_{i2}$ in (4)) with the negative real part in approximating $\hat{G}(k_ρ)$ along the paths $L_1^1$ and $L_1^2$. 

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These improvements are thought to be due to the proper choice of the path $L^1_i$ (i.e., the adequate choice of parameters $T_0$ and $t_0$) and introduction of the path $L^1_2$. That is, the accurate results observed in the spatial region far from $\rho = 0$ may be caused by the third term of (4) which is obtained through the efficient approximation of the spectral function in the path $L^1_2$ which corresponds to the SDP. On the other hand, the accurate results observed in the region near to $\rho = 0$ may be caused by the first term of (4) which corresponds to the quasi-static images.

In conclusion, an improved method for the derivation of new closed-form Green’s functions has been considered, which can give more accurate evaluation of the spatial Green’s functions than the previous method, even without the advance investigation of the spectral functions, over the wide frequency range. The combination of the present new closed-form Green’s functions and MoM may help in analyzing the electromagnetic problem relevant to the microstrip planar structures.

![Fig. 3](image.png)

Fig. 3. The numerical evaluation results for the magnitude of the vector and scalar potential Green’s functions. (Solid lines: numerical integration results, dotted lines: present results)

(a) vector potential (b) scalar potential

References


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