Synthesis of shielded parabolic acoustic reflector antennas

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1. Introduction

Nowadays reflector parabolic antennas are most widespread in atmospheric acoustics [1, 2] by virtue of the relative simplicity of their design. In this case, a parabolic reflector, which ensures a narrow gain pattern for sufficiently small overall antenna dimensions, is used to focus an acoustic beam. A protecting truncated cone shield is used to reduce the level of side lobes (LSL) and external noise as well as to protect the environment from intense acoustic radiation. Acoustic shields increase significantly the overall antenna dimensions.

A disadvantage of the previous procedures for calculating and synthesizing shielded parabolic reflector antennas suggested in [3, 6] was their specific parameters (the diameter of the reflector and the height and aperture angle of the protecting shield). This hindered significantly the synthesis of acoustic antennas. However, these works provide the basis for further investigations and development of a generalized synthesis technique.

2. Theory

Many methods of calculating electromagnetic antennas are used to calculate the parameters of acoustic antennas. Thus, to calculate the pressure field distribution across the antenna aperture, the modified Kirchhoff integral described in detail in [3-6] is used. The Fraunhofer integral is used to calculate the antenna gain pattern in the far field.

The distribution of the acoustic pressure level across the transmitting aperture of a shielded antenna (with the height \( H \) of the protective shield) is described by the modified Kirchhoff diffraction integral of the form

\[
A(R) \cdot \exp\left(j \varphi[R]\right) = \frac{2\pi R_{\text{max}}}{\pi} \int_0^\varphi A(R) \cdot \exp\left(-j \cdot k \cdot [D-H]\right) \cdot \gamma(D) \cdot R \cdot dR \cdot d\Phi,
\]

where \( A(R) \cdot \exp\left(j \varphi[R]\right) \) is the acoustic pressure field distribution across the aperture of the unshielded antenna, \( H \) is the protective shield height, \( \Phi \) is the angle counted in the aperture plane of the parabolic reflector, \( D = \sqrt{R^2 \sin^2(\Phi) + (R_s - R \cos[\Phi])^2 + H^2} \), \( \gamma(D) = \frac{1 + H/D}{2 \cdot D} \) is the coefficient taking into account a decrease in the acoustic signal amplitude with distance \( D \), \( RdRd\Phi \) is the element area of the aperture of the unshielded antenna, the factor \( \exp\left(-j \cdot k \cdot [D-H]\right) \) describes variations of the acoustic signal phase with distance \( D \), \( R_{\text{max}} = R_{\text{max}} + H \cdot \tan(\alpha) \) is the maximum radius of the transmitting aperture of the protective shield.

Thus, the initial parameters for calculating the gain pattern of the shielded antenna are the working frequency of the antenna, eccentricity of the horn gain pattern (GP), radius and focal point of the paraboloid, GP width, and relative LSL.

3. Generalized technique of synthesis of shielded reflector antennas

Calculations by the above-described technique are complicated and require long computational time and high computer power, because the amplitude and phase distributions of the pressure field across the antenna aperture corresponding to the required GP must be fitted for each particular antenna. The essence of the suggested generalized technique of synthesis of shielded reflector antennas is the following:

1. The working wavelength and the required antenna gain pattern are assigned. The radius of
the reflector and the eccentricity of the transmitting gain pattern are then calculated.

2. For the given GP, the acoustic pressure level at the transmitting aperture edge of the shield is determined from plots of three-dimensional dependences of the modulus of the gain pattern on the acoustic pressure level at the transmitting aperture edge and on the relative angular coordinates; in so doing, the amplitude distribution of the acoustic pressure field across the transmitting aperture of the shield is considered cosine on the pedestal.

3. From the acoustic pressure level at the edge of the unshielded reflector, one of the three-dimensional plots of the amplitude and phase distribution of the acoustic pressure field versus the height and radius of the transmitting aperture (or versus the aperture angle) of the protective shield is selected. The three-dimensional plots were drawn for the dimensionless parameters of the shielded antenna and different acoustic pressure levels at the edge of the unshielded reflector aperture.

4. For the plot chosen in item 3, the amplitude and phase distribution of the acoustic pressure field corresponding to the given acoustic pressure level at the edge of the transmitting shield aperture and to the gain pattern selected in item 2 are chosen.

5. The geometric parameters of the shielded antenna chosen in item 4 are de-normalized using formula (2) describing the equivalent antenna parameters. For de-normalization, the working wavelength specified in item 1 and the radius of the unshielded reflector are used.

The modified Kirchhoff’s integral is used to calculate the acoustic pressure field distribution across the transmitting antenna aperture. The equivalent representation of the antenna parameters and of the amplitude and phase distributions of the acoustic pressure field across the unshielded antenna aperture providing the basis for the suggested generalized technique of synthesis of shielded antennas is based on the normalization of the antenna parameters by the reflector radius. In so doing, \( R'_\text{max} = R_\text{max} / R_\text{max} = 1 \) is the lower aperture radius, \( F' = F / R_\text{max} \) is the focal point of the reflector, \( \lambda' = \lambda / R_\text{max} \) is the wavelength, \( R'_\text{Smax} = R'_{\text{Smax}} / R_\text{max} \) is the upper aperture radius, and \( H' = H / R_\text{max} \) is the shield height.

If all normalized parameters of two different antennas are equal, the acoustic pressure field distributions will also be equal. If the antenna parameters differ, the de-normalization is performed with the use of the following expressions:

\[
\begin{align*}
  k &= k_{\text{ref}} \cdot N, \\
  H'_{\text{ref}} &= H' \cdot N, \\
  R'_\text{Smax} &= \frac{R'_{\text{Smax}}}{R_{\text{Smax}}^{\text{ref}}} - H'_{\text{ref}} \left/ \left( N \cdot \arctan \left( \frac{N^2 - 1}{\pi^2 \left( H'_{\text{ref}} / N \right)^2} \right) \right. \right. \\
  \alpha &= \arctan \left( \tan(\alpha_{\text{ref}}) - \arctan \left( \frac{N^2 - 1}{\pi^2 \left( H'_{\text{ref}} / N \right)^2} \right) \right) \\
  \end{align*}
\]

where \( k \) is the wave number, \( k_{\text{ref}} \) is the reference wave number, \( N \) is the proportionality coefficient equal to the ratio of the reference wavelength to the working one, \( R_{\text{Smax}}^{\text{ref}} \) is the upper (transmitting) reference antenna aperture radius, \( H_{\text{ref}} \) is the height of the reference antenna shield, \( \alpha_{\text{ref}} \) is aperture angle of the reference antenna shield.

To simplify calculations, we set \( \kappa_{\text{ref}} = 1 \) and then calculate plots of the dependences of amplitude and phase distribution of the acoustic pressure field across the antenna aperture on the shield height or the shield aperture angle using Eq. (1) for different acoustic pressure levels at the aperture edges.

Figure 1 shows one of the plots of the calculated amplitude and phase distributions of the acoustic pressure field across the transmitting antenna aperture as functions of the shield height and the aperture angle. Calculations were performed for an acoustic pressure field level at the unshielded reflector edge of 0.3. Here the degree of blackening shows the phase distribution of the acoustic pressure field. The lighter regions correspond to higher values of the normalized phase counted from...
its value in the aperture center. Curves in the figure illustrate the normalized amplitude distribution of the acoustic pressure field across the shield aperture.

![Diagram of normalized amplitude and phase distributions](image)

Fig. 1. Normalized amplitude and phase distributions of the acoustic pressure field across the antenna aperture for the indicated height and aperture angle of the protective shield and an acoustic pressure level at the edge of the reflector aperture of 0.3. Here the degree of blackening shows the phase distribution of the acoustic pressure field, and curves with numbers illustrate the amplitude distribution of the acoustic pressure level across the shield aperture.

In the region where the dark zone is sharply transformed into the light zone (Fig. 1), the phase changes abruptly from the positive to negative one (a discontinuity is observed in the phase characteristic); moreover, phase discontinuities are periodic in character. Such discontinuities on the transmitting aperture are due to the diffraction of the acoustic field by large aperture edges. In general, distortions of the phase distribution across the antenna aperture result in the distortions of the antenna gain pattern; therefore, it is desirable to fit the geometric parameters of the shield so that their values were less than critical ones at which the pressure field undergoes phase discontinuities, namely, to restrict by the first half-cycle of the corresponding distribution.

Based on the results of our analysis of the calculated amplitude and phase distributions of the acoustic pressure field across the transmitting aperture, we have established that the pressure level of side lobes for any sector of the gain pattern does not exceed 35 dB if the acoustic field amplitude distribution at the edge of the transmitting shield aperture is within the limits 0.04 … 0.15. In this case, the dimensions of the shield are determined by the by the following expressions:

\[
H \approx \frac{(0.11\ldots0.56) \cdot R_{\text{max}}^2}{\lambda},
\]

\[
R_{S\text{max}} \approx (1.2\ldots1.7) \cdot R_{\text{max}} - (0.11\ldots0.56) \cdot \frac{R_{\text{max}}^2}{\lambda} \cdot \arctan \left( \frac{R_{\text{max}}^2 - \lambda^2}{\pi^2 \left( (0.11\ldots0.56) \cdot R_{\text{max}}^2 \right)} \right),
\]

\[
\alpha \approx \arctan \left( \frac{\lambda}{R_{\text{max}}} - (0\ldots0.33) - \frac{\lambda}{R_{\text{max}}} \cdot \arctan \left( \frac{R_{\text{max}}^2 - \lambda^2}{\pi^2 \left( (0.11\ldots0.56) \cdot R_{\text{max}}^2 \right)} \right) \right),
\]

and their minimum values will depend on the acoustic pressure level at the edge of the unshielded antenna.

From Eq. (4) it can be seen that the optimal height is inversely proportional to the number of wavelengths falling within the aperture radius of the unshielded antenna. That is, the larger number of
wavelengths falls within the aperture of the unshielded reflector, the greater the shield height. Moreover, from these expressions it also follows that the antenna has satisfactory properties of its gain pattern over a wide range of shield heights or working wavelengths.

4. Experimental investigations

We investigated two acoustic shielded parabolic reflector antennas, manufactured using the above-described technique, including the transmitting antenna of a system for studying acoustic wave propagation through the atmosphere and the transceiving antenna of the Zvuk-3 acoustic radar.

The parameters of the acoustic antenna for the first measuring system were calculated for the frequency band 1000–5000 Hz. The width of the gain pattern was assigned in the limits from 12 to 17°, and the level of side lobes should not exceed –25 dB (for the sector 60–90°).

Based on the foregoing, the reflector diameter of 1.5 m was chosen. From Eq. (4), we calculated that with allowance for the horn gain pattern and the acoustic pressure level at the edge of the reflector aperture equal to 0.1 at a frequency of 1000 Hz, the geometric parameters of the shield should be

\[
H = 0.3 \frac{R_{\text{max}}^2}{\lambda}, \quad R_{\text{max}} = 1.1 R_{\text{max}}.
\]

From here it follows that the height of the shield should be 0.5 m, and the diameter of the upper aperture should be 1.86 m.

In the Zvuk-3 acoustic radar, a shielded transceiving antenna with a parabolic reflector having a flat central segment and a special transmitter was used. The requirements were more stringent: the width of the gain pattern for a frequency of 3150 Hz (2000) should not exceed 10° (15°) (based on this, a reflector diameter of 0.63 m was chosen), and the level of side lobes (in the sector 60–90°) should not exceed –30 dB.

From Eq. (4) for a frequency of 3150 Hz, with allowance for the horn gain pattern, the pressure level at the reflector aperture edge, and the increased shield height due to shadowing of the upper aperture, we obtained the following parameters:

\[
H = \frac{R_{\text{max}}^2}{\lambda}, \quad R_{\text{max}} = 1.4 R_{\text{max}}.
\]

Based on the foregoing, the shield height should be 0.91 m, and the diameter of the upper aperture should be 0.88 m.

5. Conclusions

Our experimental investigations have demonstrated the correctness of the developed technique and its practical applicability. The error in determining the upper aperture radius was of the order of 5–7%. The results obtained demonstrate that shields with low heights can be used for acoustic antennas. It should be noted that this technique does not take into account the effect of aperture shadowing; therefore, experimentally measured amplitude distribution of the acoustic pressure across the shield aperture in some cases differed from the calculated ones, which distorted the antenna gain pattern in the far field. The second reason for the observed difference between the calculated and experimental data was that we disregarded the absorption coefficient of the shield walls.

6. References