Fast Iterative Algorithm for MoM Analysis of Large-scale 2-D Array Antennas

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1 Introduction

The method of moments (MoM) is one of the efficient methods for the electromagnetic analysis of array antennas. However, when the direct method such as the Gauss-Jordan method is employed to solve the matrix equation appearing in the MoM, the CPU time is proportional to the third power of the number of unknowns so that the MoM is hardly applied to the analysis model with numerous unknowns. On the other hand, large-scale array antennas, such as the array antennas in SPS (Solar Power Satellite) systems, are required to be analyzed practically. Therefore, the computational cost to solve the matrix equation has to be reduced to analyze large-scale of array antennas with a large number of unknowns.

The Gauss-Seidel iterative method can be used to solve linear equations with CPU time proportional to the second power of the number of unknowns. However, it has been pointed out that the iteration to solve the matrix equation appearing in the MoM analysis by using the conventional Gauss-Seidel iterative method usually has poor convergence. In order to improve the convergence characteristics of the iteration, a grouping technique has been proposed, which has been applied for 1-D array antennas and the convergence have been studied [6, 7].

In this research, the iterative algorithm based on the Gauss-Seidel method is applied to solve the matrix equation in the MoM analysis of 2-D array antennas. The convergence of the iterative algorithm is investigated and the effectiveness of the method is shown by some numerical examples.

2 Iterative algorithm

It has been shown that the Gauss-Seidel method can not be used directly to solve the matrix equation in MoM for analyzing the usual array antennas because the convergence criterion of the iterative method depends on the total number of the array elements, the number of the segments for each element, the array spacing, and the geometry of the antenna[6].

In the novel iterative algorithm, the antenna array is divided into a number of groups and each group consists of several neighboring array elements, so that the impedance matrix can be decomposed into a number of sub matrices corresponding to the group of the array elements. The diagonal sub matrices in the impedance matrix describe the self and mutual impedance between the divided unknown segments in the same group and the off-diagonal sub matrices include the mutual impedance between two divided unknown segments of different groups. The iterative unit is then changed to the sub matrices, and the sub matrices are the basic iteration units rather than the matrix element in the ordinary Gauss-Seidel iteration method. If the array antenna contains N elements, each element is divided into M dipole segments for sub domain

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analysis and each group consists of K array elements, the iterating procedure is expressed by:

$$[\bar{I}]_i^{(l_s+1)} = [\bar{Z}]_{ii}^{-1}\left[[\bar{V}]_i - \sum_{j=1}^{i-1}[\bar{Z}]_{ij}[\bar{I}]_j^{(l_s+1)} - \sum_{j=i+1}^{N/K}[\bar{Z}]_{ij}[\bar{I}]_j^{(l_s)}\right]^T, i = 1 \sim N/K;\ l_s = 0 \sim L_s. \quad (1)$$

where $[\bar{I}]_i$ is a (MK) current vector of the group i, $[\bar{V}]_i$ is the voltage vector of group i, and $[\bar{Z}]_{ij}$ is a $(MK)\times(MK)$ matrix, whose elements are the self and mutual impedance between the segments of two groups i and j. The inverse matrix $[\bar{Z}]_{ii}^{-1}$ is evaluated by using a direct method such as the Gauss-Jordan method. Notation $[\cdot]^T$ indicates the transposition of the matrix. For fast convergence, the initial $[\bar{I}]_i^{(0)}$ is assumed to be the current on a single group of the array elements ignoring the mutual coupling between the groups [6].

3 Numerical examples

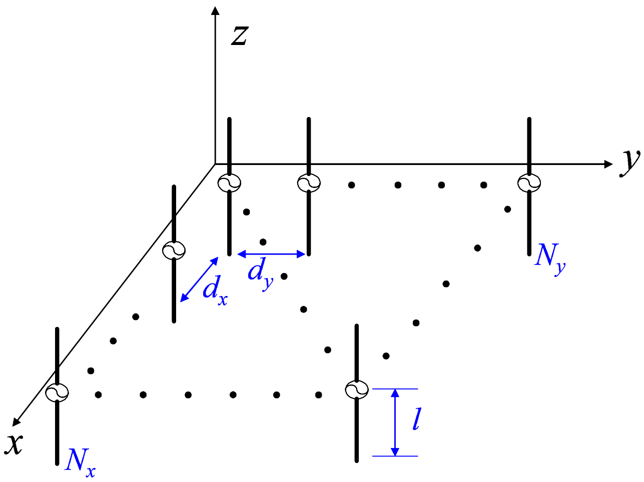

Figure 1: 2-D dipole array with $N = N_x \times N_y$ elements.

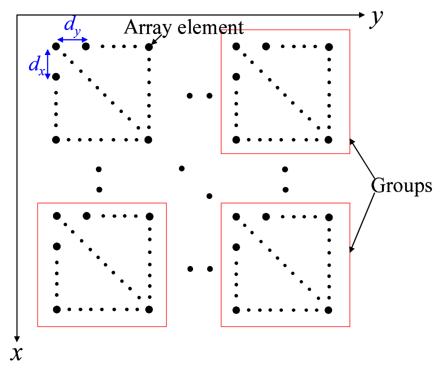

Figure 2: Each group contains severval neigbouring array elements.

MoM analysis of a 2-D dipole antenna array is performed by using the present method. Fig. 1 shows the array geometry with $N_x \times N_y$ array elements, where d_x, d_y are array spacing and each dipole has length of $2l$. The Galerkin 's method and wire-grid modeling are applied for the MoM analysis [8]. Fig. 2 shows how to make neighbouring groups with each group consisting of K array elements.

Fig. 3 shows the required iteration steps versus array spacing when M=3 and N_x=N_y=32. It is found that when K increases, the required iteration steps becomes small, which indicates the grouping technique makes the iteration much more stable especially when the array spacing is small. The singularity at $d_x/\lambda = d_x/\lambda = 0.6$ in the curve of K=16 is due to the strong coupling between the groups which appears periodically with increase of the array spacing.

Fig. 4 shows the CPU time required to perform the iteration. The value of the CPU time was measured by using a Pentium-4 3.2GHz PC. Although a large K can reduce the iteration steps to decrease the CPU time, a too large K would result in consuming a longer CPU time on the contrary because CPU time for evaluating $[\bar{Z}]_{ii}^{-1}$ is also required which is proportion to K to the third power. It can be concluded from the figure that a proper K value can greatly reduce the CPU time, while keeping the iteration process stable.

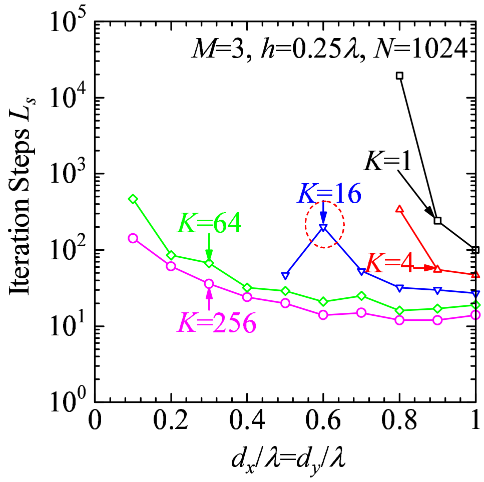

Figure 3: Required iteration steps versus array spacing.

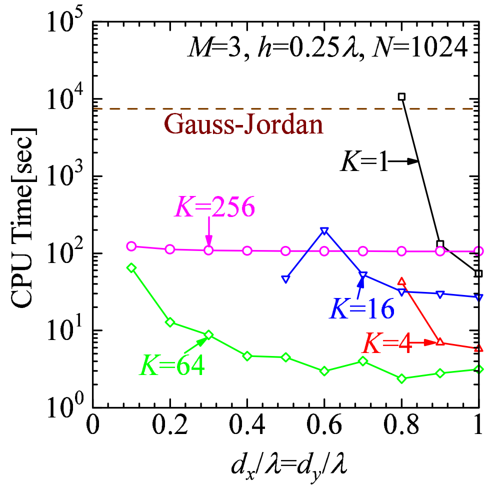

Figure 4: CPU time versus array spacing.

The error of I_L at the final iteration step L_s is estimated by the residual norm, which is defined by

$$\Pi_L = \frac{||ZI_L - V||}{||V||}. \tag{2}$$

This quantity is evaluated for various K and shown in Fig. 5, where the value of L_s is the same as shown in Fig. 3. The residual norm is smaller than 5×10^{-9} for all cases, which indicates that good accuracy of the iteration is obtained.

In order to show the validity of this method, the CPU time for solving the matrix equation versus N is shown in Fig. 6. The curve of the Gauss-Jordan method, one of the traditional iteration methods, is also plotted in the same figure for comparison. As expected, the CPU time is proportional to N^3 by using the Gauss-Jordan method, while it is nearly proportional to N^2 by using the present method with a proper K. The cost saving effect of the numerical computation is significant.

4 Summery

The iterative algorithm based on the Gauss-Seidel method has been applied to solve the matrix equation in the MoM analysis of 2-D array antennas. A 2-D dipole antenna array has been analyzed as an example to show the convergence characteristics and the validity of the algorithm. The algorithm requires the CPU time of order of N^2 approximately when the grouping technique is properly used, which is expected to be applied to the MoM analysis of practical large-scale array antennas efficiently.

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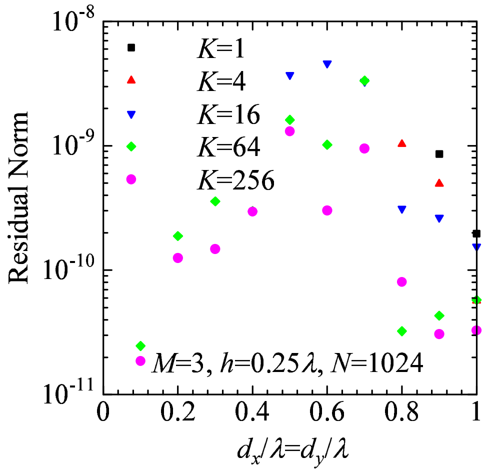

Figure 5: Residual norm versus array spacing.

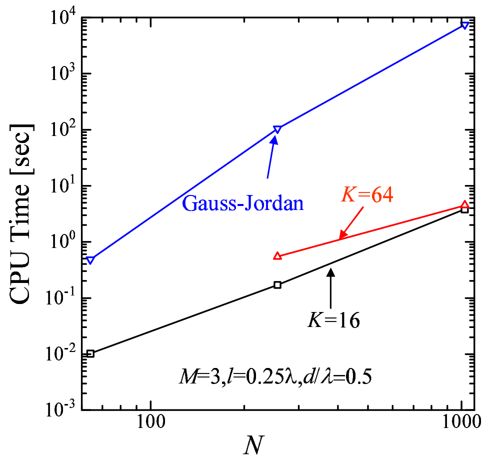

Figure 6: CPU time versus total number of array elements.

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