1. Introduction

As a promising technique for high data-rate transmission, Orthogonal Frequency Division Multiplexing (OFDM) is being considered for many emerging wireless applications. In fact, it has been successfully used in many environments, such as DAB, DVB, and HiperLAN-II. In a classical OFDM system, the entire bandwidth is divided into many orthogonal subcarriers, where information symbols are transmitted in parallel over these subcarriers with long symbol duration in order to deal with frequency-selective fading of wireless environments. The overlapping spectrum of the subcarriers in OFDM system requires an accurate frequency recovering system. However, in time variant mobile radio environment, the relative movement between transmitter and receiver resulted in frequency offset due to Doppler frequency shifts, hence the carriers cannot be perfectly synchronized. This imperfection destroys orthogonality among subcarriers and causes intercarrier interference (ICI) to occur in addition to rotation and attenuation. Furthermore, the degradation of BER performance increases rapidly with increasing frequency offset occurrence in OFDM system [1].

Several methods have been proposed to reduce the effect of the ICI. One of the methods is frequency-domain equalization [2]. Time-domain windowing is another way to reduce the effect of frequency offset [3]. The ICI suppression in multiple-input multiple-output (MIMO) OFDM is studied in [4]. A self-ICI-cancellation approach has been proposed, which transmits each symbol over a pair of adjacent or non-adjacent subcarriers with a certain phase shift [5, 6, 7]. This method can suppress the ICI significantly with a reduction in bandwidth efficiency. In single-carrier systems, partial response signaling has been studied to reduce the sensitivity to time offset without sacrificing the bandwidth [8]. In the frequency domain, the partial response with correlation polynomial \( F(D) = 1 - D \) was used to mitigate the ICI caused by carrier frequency offset [9]. The optimum weights for partial response coding that minimize the ICI power were derived [10]. However, by using polynomial coefficients with integer values reduces the complexity of the receiver. In this paper, we study the effect of partial response OFDM (PR-OFDM) system with integer polynomial coefficients and symbol-by-symbol suboptimum detection technique. Besides than ICI, the effect of this method on the PAPR is also investigated.

This paper is organised as follows. In Section 2 we describe a PR-OFDM system. The ICI and PAPR expressions and analysis is included in Section 3. Then, in Section 4, the simulation results are presented to demonstrate the performance of PR-OFDM systems with integer polynomial coefficients.

2. Partial Response OFDM system (PR-OFDM)

The baseband model of PR-OFDM is shown in Fig.1. At the transmitter, the modulated signal is encoded by partial response polynomial. Precoding is also performed before modulation in order to avoid error propagation during decoding process. Let \( X_k \) be the symbols to be transmitted and \( c_i \) be the
coefficients for partial response polynomial, the transmitted signal at the $k$-th subcarrier can be expressed as

$$ S_k = \sum_{i=0}^{K-1} c_i X_{k-i} $$

where $K$ is the number of coefficients or length of the polynomial. Without loss of generality, $E[x_k] = 1$ and $E(x_k x_j^*) = 0$ for $k \neq j$ is assumed.

The transmitted OFDM signal in time domain is

$$ y(t) = \sum_k S_k e^{j2\pi f_k t}, \quad 0 \leq t \leq T_s $$

where $f_k = f_0 + k \Delta f$ is the frequency of the $k$-th subchannel, $\Delta f = 1/T_s$ is the subchannel spacing, and $T_s$ is the symbol duration. The coded signals can be recovered by a maximum-likelihood (ML) sequence detector at the receiver. After passing through a time-varying channel with the impulse response $h(t, \tau)$, the received signal is

$$ \tilde{y}(t) = \int h(t, \tau) y(t-\tau) d\tau $$

The channel impulse response for the frequency-selective fading channel can be described by

$$ h(t, \tau) = \sum_{l=1}^{v} h_i(t) \delta(t - \tau_i(t)) $$

$v$ is the total number of non-zeros taps in the channel response, $h_i(t)$ represents the time variant attenuation factor of the $l$-th path and $\tau_i(t)$ is the time varying delay of $l$-th path. The channel impulse response can also be represented as :

$$ h(t, \tau) = \sum_{l=1}^{v} h_i(t) \text{exp}\left( j 2\pi f_{D, l}(t) \tau \right) \delta(t - \tau_i(t)) $$

where $h_i(t)$ changes caused by movement of mobile receiver, which is known as the Doppler frequency shifts, $f_{D, l}(t) t$.

3. ICI and PAPR Analysis of PR-OFDM

The instantaneous envelope power of the signal is the real-valued function. The PAPR of the transmitted OFDM signal, $y(t)$ can be defined as

$$ \text{PAPR} = \max \left\{ \frac{\left| y(t) \right|^2}{E\left[ \left| y(t) \right|^2 \right]} \right\} $$

where $E[\left| y(t) \right|^2]$ denotes the expectation of the average power value. The received signal can be written as

$$ \tilde{y}(t) = h(t, \tau) * y(t) + z(t) $$

where $z(t)$ is the additive white Gaussian noise (AWGN). The output of the DFT at the receiver for a time-block $\{-v-1, \ldots, N-1\}$ where $N$ is the number of carriers can be written as
\[ Y(p) = G(p, p)S(p) + \sum_{q=0}^{N-1} G(q, p)S(q) + Z(p) \]  

(9)

for \( p=0, \ldots, N-1 \). The \( G(p, p)S(p) \) gives the desired signal value for subcarrier \( p \) with an average carrier power of \( E[|G(p, p)S(p)|^2] \). While \( G(q, p) \) is defined as the subcarrier frequency offset response for the \( p \)-th subcarrier \([5]\). It is also the ICI effect of the \( q \)-th subcarrier to the \( p \)-th subcarrier with the occurrence of normalized frequency offset, \( \varepsilon \). In the case of time-variant the equation \( G(p, q) \) becomes

\[
G(q, p) = \frac{1}{N} \sum_{n=0}^{N-1} h_n \exp(j \theta_n) \exp\left(j \frac{2\pi}{N} (p-q)\right) \exp\left(-\frac{2\pi}{N} n \varepsilon\right)
\]  

(10)

By letting \( c_i \) be the coefficients for partial response and \( K \) is the number of coefficients or length of partial response, the transmitted signal can be expressed as in Eq.(1). Therefore, the ICI power on the \( m \)-th subcarrier also can be expressed into

\[
P_{ici}(m) = E\left[\sum_{l=0}^{N-1} G(l)S(m-l)\right]^2
\]  

(11)

In terms of the partial response coding coefficients, it can be expressed as

\[
P_{ici}(m) = \sum_{m-l=i}^{K-1} E(\tilde{\alpha}m-l-i)^2 \sum_{m-l=i}^{K-1} \sum_{k=0}^{K-1-k} \sum_{l=0}^{K-1-l-k} c_{ijkl} E(\tilde{\alpha}m-l-i) \tilde{\alpha}m-l-i-k)
\]  

(12)

4. Results

Here we present our simulation results for the performance of PR-OFDM system with integer polynomials. The number of subcarriers used in this simulation is 128 subcarriers. Different polynomial length (up to \( K=4 \)) and coefficients are investigated. The coefficients are limited to the value of \( \pm 1 \) and 0. At each polynomial length, the polynomial that gives the lowest carrier to interference ratio (CIR) is chosen. Our simulation is conducted with the assumption of flat fading channel. Symbol-by symbol suboptimum detection is used at the receiver.

In the PAPR distribution analysis, the complementary cumulative distribution function (CCDF) of an OFDM signal for a given PAPR level, \( X \) dB, is the probability that the PAPR of the OFDM frame exceeds a certain threshold \( X \) dB. This is defined as \( \text{Prob}(\text{PAPR} > X \text{ dB}) \). Fig.2 shows the PAPR for \( N=64 \). The PAPRs of PR-OFDM system at different number of coefficients is compared to normal OFDM system without partial response. At \( 10^{-3} \) probability, \( K=3 \) improves the PAPR by 0.5 dB and 2 dB in contrast to when the number of coefficients is 4 and 2 respectively.

![Fig. 2. PAPR performance of the PR-OFDM system](image)

![Fig. 3. CIR comparison of PR-OFDM with respect to constant frequency offset](image)
The CIR performance of PR-OFDM is shown in Fig.3. We can observe clearly from this figure that the PR-OFDM has around 12 dB improvement when \( \varepsilon = 0.2 \) as compared with that of a conventional OFDM system. As shown from Fig.3, \( K = 3 \) and \( 4 \) gives slightly higher and improved CIR compared to when \( K = 2 \) is used in the polynomial. The PR-OFDM improves the BER of ordinary OFDM system with the presence of frequency offset, due to Doppler shift in the channel is shown in Fig.4. The higher degree of polynomial has a lower BER floor at high SNR value. The BER performance and error floor for \( K = 2 \) and \( 3 \) are almost similar. However, when \( K = 4 \), the error floor is reduced from 0.0051 to 0.0026.

5. Conclusion

In this paper, PR-OFDM system has been studied. Polynomial coefficients with integer values are used to reduce the complexity of the receiver. ICI is deliberately introduced in a controlled manner through the polynomial functions. The effectiveness of PR-OFDM system with integer polynomial coefficients on the ICI and PAPR is investigated. From the results obtained, PAPR is reduced with PR-OFDM compared to without the partial response function. About 12 dB improvement in CIR is shown and better error floor performance is obtained. Therefore, PR-OFDM with integer polynomial coefficients gives a solution to adverse the effects of both ICI and PAPR in OFDM systems simultaneously and in a simple manner. This system is feasible and can be applied in future broadband system development such as MIMO-OFDM system.

REFERENCES


