Investigation of an Arbitrarily Rotated Slot on a Conducting Spherical Cavity

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Abstract

The dyadic Green’s function formulation for analyzing an arbitrarily rotated rectangular slot antenna is presented. The slot can be arbitrarily rotated and located on a conducting spherical cavity. Some radiation pattern characteristics of the antenna are investigated and also pointed out when the slot length and cavity size are changed. Finally, numerical results of the derived dyadic Green’s function approach are compared with that of the approximate vector potential one.

1. INTRODUCTION

Investigations of slots involving spherical geometries have been considered for many years. Some of them are based on horizontal rectangular and zonal slots on a conducting spherical cavity [1]-[3]. Other advanced works involving dielectric resonator antennas (DRA’s) have also been reported for millimeter-wave applications [4]-[5]. This kind of antenna configuration is interesting because the slot arrays can be directly fabricated on its spherical geometry in many styles of orientations synthesizing various styles of radiation patterns. In this paper, the analytical formulas are achieved by using the dyadic Green’s function to predict electromagnetic wave radiating far from the conducting spherical cavity. It also depicts some results of the radiation properties of an arbitrary-rotated rectangular slot. The radiation patterns in three principle planes cut on 3-D patterns are also shown. The results can be useful in further works for synthesizing arbitrary radiation patterns when arrays of other styles of slot orientations are adapted.

2. GEOMETRY OF THE PROBLEM

In Fig. 1, the geometry of the problem is shown, where \( a \) is the radius of the conducting spherical cavity. The slot angle, \( \Theta_s \), is the location of the slot along the latitude of the cavity. The length and width of the slot are \( 2l \) and \( w \), respectively. The coordinate, \( P(r, \Theta, \phi) \), is the observation point far from the antenna structure. The source point locating at the center of the slot is denoted by the coordinate, \( P_c(r_c, \Theta_c, \phi_c) \). The formulation of this antenna configuration is being discussed in the next section.

3. ANALYTICAL FORMULATION

The electric field radiates from a horizontal slot, locating at \( \phi = 0^\circ \) and \( \Theta = 90^\circ \) on the conducting spherical cavity, for a magnetic current \( \overrightarrow{M}(\vec{r}) \) as the source are related to the electric dyadic Green’s function [6] by

\[
E(\vec{r}) = \int_{\text{SA}} \overrightarrow{G_{EM}}(\vec{r}, \vec{\rho}) \cdot \overrightarrow{M}(\vec{\rho}) \, dS'.
\]

The notation \( \overrightarrow{G_{EM}} \) indicates the electric dyadic Green’s function with the magnetic current source. The integral (over \( SA \)) holds for the entire slot area. The electric dyadic Green’s function with a magnetic current source is given by

\[
\overrightarrow{G_{EM}}(r, \Theta, \phi; r', \Theta', \phi') = \frac{e^{jkr'}}{4\pi} \sum_{\mu = -\infty}^{\infty} \sum_{\nu = -\infty}^{\infty} \overline{C_{\mu \nu}}(f^{\mu \nu}) \times \left[ P(k) + b_k \overrightarrow{P^{(2)}(k)} \right] \left[ \mu L_{\mu}^{\cos} (m\phi) \hat{\theta} + L_{\mu}^{\cos} (m\phi) \phi \right] + \left[ N(k) + a_k \overrightarrow{N^{(2)}(k)} \right] \left[ \mu L_{\mu}^{\sin} (m\phi) \hat{\theta} - L_{\mu}^{\sin} (m\phi) \phi \right].
\]
where the vector wave functions \( \mathbf{M}(k), \mathbf{M}^{(1)}(k), \mathbf{N}(k) \) and \( \mathbf{N}^{(1)}(k) \) are defined in [6]. The factors involving associated Legendre functions, \( L_1 \) and \( L_2 \), are defined as

\[
L_1 = \frac{m}{\sin(\theta)} P^m_0(\cos \theta) \quad \text{and} \quad L_2 = \frac{1}{\partial \theta} P^m_0(\cos \theta),
\]

while the \( L_1' \) and \( L_2' \) are defined for coordinates of the source point (i.e. \( \theta' \)). In the above equations, \( j \) is an imaginary unit while the term, \( j \phi(r) \), is the first-kind spherical Bessel function of order \( n \). The term \( \delta_{01}(r) \) denotes the second-kind spherical Hankel function of order \( n \) and \( P^m_0(\cos \theta) \) is an associated Legendre of the cosine function.

At the first stage, if the voltage distribution along the slot’s length can be assumed to be sinusoidal as

\[
V(\phi') = \begin{cases} 
V \sin \left( \frac{2\pi}{\lambda} (1 - a \phi') \right) & |\phi'| < \frac{1}{a} \\
0 & |\phi'| > \frac{1}{a}
\end{cases}
\]

Then the magnetic current, \( \mathbf{M}(\phi') \), along the slot can equivalently be found from

\[
\mathbf{M}(\phi') = -\mathbf{n} \times \mathbf{E}(\phi')
\]

where \( \mathbf{n} \) and \( \mathbf{E}(\phi') \) are a unit vector normal to the surface of the problem and the electric field distribution at the aperture of the slot, respectively. By using (3) and (4) it is found that

\[
\mathbf{M}(\phi') = -V \sin \left( \frac{2\pi}{\lambda} (1 - a \phi') \right) \mathbf{\hat{\phi}} \left\{ -\frac{1}{a} < \phi' < \frac{1}{a} \right\} \left( \theta - \alpha < \theta' < \theta + \alpha \right)
\]

where \( \alpha \) is the incremental angle extend up and down the slot angle \(( \phi )\) exposing the width \(( w )\) of the slot. When substitute (5) into (1), one gets the expression of the form

\[
\mathbf{E}(\phi') = \iint \mathbf{G}_{EM}^{\phi}(\mathbf{r}, \mathbf{r}') j \mathbf{\hat{\phi}} + \mathbf{G}_{EM}^{\phi}(\mathbf{r}, \mathbf{r}') j \mathbf{\hat{\phi}} M(\phi') dS',
\]

where \( dS' = r'^2 \sin \theta' d\theta' d\phi' \) is the incremental surface of the slot. The \( \theta \)-component and \( \phi \)-component electric dyadic Green’s functions are given, respectively, by

\[
\mathbf{G}_{EM}^{\phi}(\mathbf{r}, \mathbf{r}') = \frac{e^{-j\beta r}}{4\pi r} \sum_{m=-n}^{n} \sum_{n=0}^{\infty} C_{m,n} k^{j m} \left\{ \cos(m\phi') \cos(m\phi) + \sin(m\phi) \sin(m\phi') \right\}
\]

\[
\begin{align*}
&\left\{ -L_1 L_2 j_0(kr) + h_0^{(2)}(kr) \right\} + j L_1 \frac{1}{kr} \frac{\partial}{\partial(kr')} \left[ k r' j_n(kr) \right] \\
&+ a_n \frac{1}{kr} \frac{\partial}{\partial(kr')} \left[ k r' h_n^{(2)}(kr) \right]
\end{align*}
\]

and

\[
\mathbf{G}_{EM}^{\theta}(\mathbf{r}, \mathbf{r}') = -\frac{e^{-j\beta r}}{4\pi r} \sum_{m=-n}^{n} \sum_{n=0}^{\infty} C_{m,n} k^{j m} \left( j^{m+1} \right),
\]

\[
\left\{ \cos(m\phi') \sin(m\phi') - \sin(m\phi) \cos(m\phi') \right\}
\]

\[
\begin{align*}
&\left\{ -L_1 L_2 j_0(kr) + h_0^{(2)}(kr) \right\} + j L_1 \frac{1}{kr} \frac{\partial}{\partial(kr')} \left[ k r' j_n(kr) \right] \\
&+ a_n \frac{1}{kr} \frac{\partial}{\partial(kr')} \left[ k r' h_n^{(2)}(kr) \right]
\end{align*}
\]

By expanding (6) as functions of \( V_m \) integrating over the entire slot area and rearranging terms at \( r' = a \), one can obtain the final expressions as

\[
E_\theta(r, \theta, \phi) = \frac{e^{-j\beta r}}{4\pi r} \sum_{m=-n}^{n} V_m (2 - \delta_0) \frac{2n+1}{n(n+1)(n+m)!} \left( \begin{array}{c} \frac{k a}{n+1}(j^{m+1} \sin(m\phi)) \\
\left\{ j L_1 B_1 \left[ m P^m_0(\cos \theta) \right] d\theta' \\
- L_1 B_2 \int_{\theta-a}^{\theta+a} d\phi' \left[ m P^m_0(\cos \phi') \sin \phi' \sin \theta \right] \right\} 
\end{array} \right),
\]

\[
E_\phi(r, \theta, \phi) = \frac{e^{-j\beta r}}{4\pi r} \sum_{m=-n}^{n} V_m (2 - \delta_0) \frac{2n+1}{n(n+1)(n+m)!} \left( \begin{array}{c} \frac{k a}{n+1}(j^{m+1} \sin(m\phi)) \\
\left\{ j L_2 B_1 \left[ m P^m_0(\cos \phi) \right] d\phi' \\
- L_1 B_2 \int_{\theta-a}^{\theta+a} d\phi' \left[ m P^m_0(\cos \phi') \sin \phi' \sin \theta \right] \right\} 
\end{array} \right).
\]

In (9) and (10), the terms, \( B_1 \) and \( B_2 \), are defined as

\[
B_1 = -\frac{1}{ka} \frac{\partial}{\partial(ka)} \left[ ka j_n(ka) \right] + a_n \frac{1}{ka} \frac{\partial}{\partial(ka)} \left[ ka h_n^{(2)}(ka) \right]
\]

and

\[
B_2 = j_n(ka) + b_n h_n^{(2)}(ka).
\]

The modal coefficients, \( a_n \) and \( b_n \), are given by

\[
a_n = -\frac{j_n(ka)}{h_n^{(2)}(ka)} \quad \text{and} \quad b_n = \frac{(d/dka)j_n(ka)}{(d/dka)h_n^{(2)}(ka)}.
\]

The modal voltage distribution function, \( V_m \), can be expanded and described by

\[
V_m = \begin{cases} 
\frac{V}{\sqrt{a}} \sin(\pi k a), & m = 0 \\
2V \frac{ka}{(ka)^2 - m^2} \left( \begin{array}{c} \cos \left( \frac{m\pi}{a} \right) - \cos(k a) \\
\sin(ka) \end{array} \right), & m \neq 0, m \neq ka
\end{cases}
\]

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where \( V \) is a constant amplitude of the voltage source.

The results obtained in (9) and (10) are for a horizontal slot that is locating at \( \phi = 0^\circ \) and \( \theta_s = 90^\circ \) only, as shown in Fig. 2 (i.e. on the x-axis). To achieve the arbitrary-rotated slot as shown in Fig. 2 (i.e. on the \( x^* \)-axis), the rotation of the spherical coordinate system can be applied.

![Diagram of arbitrary-rotated slot](image)

From [7], the rotation matrices about x-axis and y-axis by the angle \( \gamma \) and \( \theta_s \), respectively, can be defined as

\[
[R_x] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \gamma & \sin \gamma \\ 0 & -\sin \gamma & \cos \gamma \end{bmatrix} \quad \text{and} \quad [R_y] = \begin{bmatrix} \sin \theta_s & 0 & \cos \theta_s \\ 0 & 1 & 0 \\ -\cos \theta_s & 0 & \sin \theta_s \end{bmatrix}.
\]

By transforming the rotation matrices into spherical coordinate system, one can find relations between the original coordinates and the rotated ones as follows:

\[
\begin{bmatrix} r \sin \theta' \cos \phi' \\ r \sin \theta' \sin \phi' \\ r \cos \theta' \end{bmatrix} = [R_x][R_y]\begin{bmatrix} r \sin \theta \cos \phi \\ r \sin \theta \sin \phi \\ r \cos \theta \end{bmatrix}.
\]

After solving (14), the relations of \( \theta' \) and \( \phi' \) can be obtained. To transform a vector quantity, such as an electric field, the rotated unit vectors must also be determined. The rotated unit vectors can be found by solving

\[
[u_s']^{-1} = \begin{bmatrix} \sin \theta' \cos \phi' & \sin \theta' \sin \phi' & \cos \theta' \\ \cos \theta' \cos \phi' & \cos \theta' \sin \phi' & -\sin \theta' \\ -\sin \phi' & \cos \phi' & 0 \end{bmatrix}.
\]

By solving (16) for the rotated unit vectors, the final form of the electric field rotations can be shown as

\[
\begin{bmatrix} E_{\phi} \\ E_{\theta} \end{bmatrix} = \begin{bmatrix} X_1 & X_2 \end{bmatrix} \begin{bmatrix} E_{\phi} \\ E_{\theta} \end{bmatrix},
\]

where

\[
X_1 = \frac{\cos \theta \sin \phi \cos \gamma - \cos \phi \sin \gamma}{\sin \theta} \quad \text{and} \quad \sin \theta \quad \sin \theta \quad \sin \theta \quad \sin \theta
\]

\[
X_2 = \frac{-\cos \theta \sin \gamma \sin \phi + \cos \gamma (\cos \theta \cos \phi \cos \theta_s + \sin \theta \sin \theta_s)}{\sin \theta}.
\]

In deriving of (13) to (14), it should be noted that the assumption of far-zone field is adopted (i.e. \( r \to \infty \) and \( r^* \approx r \). The electric field components \( E_{\phi}' \) and \( E_{\theta}' \) are similar to (9) and (10), respectively, except that the coordinate parameters \( \theta \) and \( \phi \) are changed to be \( \theta' \) and \( \phi' \), respectively. Additionally, if the rotation about the z-axis is required, simply replaced the angle \( \phi \) by \( \phi - \beta \) where \( \beta \) is the angle of rotation about the z-axis.

4. Numerical Results

The numerical results of total electric field radiates from the antenna are obtained by summing modal terms \( (n, m) \) in (9) and (10). Since the calculations cannot be carried out to infinity, the convergence tests had been studied by varying the radius of the cavity and positions of the slot. It can be concluded that the summation of modal terms of \( ka \approx 8 \) are enough to get the converged directivity results within four digits of precision, where \( k \) is the wave number.

Firstly, to verify the formulation of the rotated slot, the result of 3-D radiation pattern for a horizontal slot, Fig. 3(a), is compared with the rotated ones (typically shown in Fig. 3(b)). The results are for the case of \( ka = 2 \), \( l = \lambda/4 \) and \( w/2 = \alpha = \lambda/10^4 \). As can be seen in Fig. 3(b), it is apparent that the rotated slot result is indeed rotated about the x- and y-axis by the angle \( \gamma = 45^\circ \) and \( \theta_s = 45^\circ \). Various values of \( \gamma \), \( \theta_s \) and \( ka \) have been tested and led to the same conclusion that the main beams are...
correctly rotated to the specified angles, $\gamma$ and $\theta_s$, as expected.

From Fig. 4, it is seen that the maximum directivities are high for small $\theta_s$ while the slot width is the same as before. From Fig. 4, it is expected that the maximum directivities monotonically decrease and converge to some constant values. The total length of the slot is $2/l$. Some investigations on maximum directivities by varying slot length and cavity size have been considered as shown in Fig. 4, where $C_f$ is defined as the slot length factor so that the total length of the slot is $2/l C_f$. The results are obtained by varying the cavity size ($ka$) from 2 to 10 and $C_f$ from 2 to 12, while the slot width is the same as before. From Fig. 4, it is seen that the maximum directivities are high for small $ka$ and $C_f$ (i.e. $C_f$ about 2 to 4). These happened because the total slot’s lengths exceeded $1\lambda$ for that range causing split-beam patterns. However, for large $C_f$ and $ka$, it is noticeable that the maximum directivities monotonically decrease and converge to some constant values.

Finally, to examine the accuracy of the dyadic Green’s function, the radiation patterns for (a) xy- and (b) xz-plane ($\theta_s = 90^\circ$, $2l = \lambda/4$, $ka = 10$) are compared with that of the vector potential approach in Fig. 5. The main beams of both approaches are in good agreement. However, the minor lobes are different because the dyadic Green approach also predicts the scattered field too. From [8], it was concluded that the dyadic Green’s results were closer to the experiment ones.

5. Conclusion

The dyadic Green’s function is used to analyze an arbitrary-rotated rectangular slot antenna by using the rotations of coordinate system. From the results it can be concluded that the final formulas can be used to rotate the electric field correctly. The investigations on the maximum directivities are also considered. The results show that for small $ka$ and $C_f$, the maximum directivities are high and rather unpredictable. However, for large $C_f$ and $ka$, the directivities monotonically converge to their constant values. These studies can be further applied for radiation pattern and polarization synthesizing.

Finally, some of the results from the dyadic Green’s function approach are examined and compared with that of the approximate vector potential approach. Both methods are in good agreement at the main beams of the radiation patterns. It can be observed that the dyadic Green’s function approach can predict the scattered field better than the vector potential approach [8] when compared with the experimental results.

References