Validation of Gain Measurement in the Liquid Based on Extended Friis Transmission Formula

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1. Introduction

Because the higher frequency band will be used for the mobile communication devices, it is required to establish the technique of calibrating the electric probes for their SAR (Specific Absorption Rate) assessment, especially over 3GHz. For the conventional calibration with rectangular waveguide system commonly used in 300MHz–3GHz [1], the tip size of the probe cannot be ignored so that the calibration is more inaccurate as the frequency is higher. The authors have developed an alternative calibration without using the waveguide system [2]–[4].

For our calibration, first, the far-field gain of the reference antenna operated in the tissue-equivalent liquid is measured, and then the electric field radiated by the antenna is related to the output voltage of the electric probe [1]. For the gain measurement, the \( S \) parameters between the two antennas with linear alignment are measured, and the far-field gain is determined by use of the two-antenna method based on the Friis transmission formula in the conducting medium [2], [5]. The decay of the fields in the liquid [1], [3] is too large to receive the signal radiated by the transmitting antenna in its far-field region. For higher frequency over 3GHz, this nature makes the transmittable distance shorter in terms of the wavelength in the liquid. To overcome this difficulty, the authors have examined the technique of estimating the far-field gain of the antenna in the liquid by using \( S_{21} \) measured in the near-field region and curve-fitting based on extended Friis transmission formula in the conducting medium that we proposed [3], [4].

In this paper, the dynamic range of the measurement system is enhanced by inserting the amplifier in front of the transmitting antenna and increasing its input power to measure \( S_{21} \) in the far-field region with a vector network analyzer. Then, the far-field gain, that is, the absolute gain estimated by using \( S_{21} \) measured in the near-field region is compared with that by using \( S_{21} \) measured in the far-field region. This fact leads to the validity of our technique of estimating the far-field gain of the antenna in the liquid.

2. Principle of Measurement

2.1 Far-Field Gain Estimation Using \( S_{21} \) in the Far-Field Region

As shown in Fig. 1, two antennas are aligned in the liquid. Port 1 and 2 are defined as the ports of the two antennas. If the distance between the two antennas, \( r \), is sufficiently large, the power transmission can be described as

\[
|S_{21}(r)|^2 = (1 - |S_{11}|^2)(1 - |S_{22}|^2)|G_1||G_2|\frac{|G_1||G_2|}{(2\pi r)^2} \exp(-2\alpha r),
\]

(1)

where \( S_{ii} \) is denoted as the reflection coefficient at port \( i \) and \( |G_i| \) is denoted as the far-field gain of the antenna connected to port \( i \). (1) is valid in the far-field region of the antenna because it can be derived from the conventional Friis transmission formula which is valid in the far-field region. The magnitude of \( S_{21} \) in dB representation can be expressed as

\[
|S_{21}|_{dB} = A - 20 \log_{10} r - 8.686 \alpha r,
\]

(2)
where the sum of the far-field gain in dB representation is given as
\[ G_{1,\text{dB}} + G_{2,\text{dB}} = A + 20 \log_{10}(2\beta) - 10 \log_{10}(1 - |S_{11}|^2) - 10 \log_{10}(1 - |S_{22}|^2). \]  
(3)

If the two antennas are identical, that is, \( G_{1,\text{dB}} = G_{2,\text{dB}} = G_{\text{dB}} \), the far-field gain can be obtained as
\[ G_{\text{dB}} = 0.5A + 10 \log_{10}(2\beta) - 5 \log_{10}(1 - |S_{11}|^2) - 5 \log_{10}(1 - |S_{22}|^2). \]  
(4)

When \( |S_{21}| \) can be measured as a function of the distance, \( r \), and fitted to the curve given by (2), the constant, \( A \), and the attenuation constant, \( \alpha \), are determined. The phase of \( S_{21} \) is assumed to be proportional to the distance, \( r \), and is given as
\[ \angle S_{21} = -\beta r + B, \]  
(5)

where \( B \) is a constant. When \( \angle S_{21} \) can be measured as a function of the distance \( r \) and fitted to the line given by (5), the phase constant, \( \beta \), is determined. Then, the far-field gain can be evaluated by substituting the values of the constants \( A \), \( \alpha \) and \( \beta \) to (4).

2.2 Far-Field Gain Estimated by Curve Fitting for Near-Field Gain

The authors extended the formula (1) and examined the technique of estimating the far-field gain of the antenna in the liquid by using \( S_{21} \) measured in the near-field region [3], [4], as
\[ |S_{21}(r)|^2 = (1 - |S_{11}|^2)(1 - |S_{22}|^2) \frac{G_1G_2}{(2\beta r)^2} \exp(-2\gamma r) \exp\left(\frac{c_1}{r} + \frac{c_2}{r^2} + \cdots\right), \]  
(7)

where \( \gamma = \alpha + j\beta \) is the propagation constant in the liquid, and \( c_1, c_2, \cdots \) are complex constants which is dependent upon antenna’s configuration and surrounding medium. Because the two antennas are identical, that is, \( |G_1| = |G_2| = |G| \), the near-field gain can be also expressed as
\[ G_F(r) = |G| \exp\left(\frac{c_1}{r} + \frac{c_2}{r^2} + \cdots\right). \]  
(8)

The corresponding dB representation can be approximated as [4]
\[ G_{F,\text{dB}}(r) = G_{\text{dB}} + \frac{C_1}{r} + \frac{C_2}{r^2}. \]  
(9)

where \( C_1 \) and \( C_2 \) are real constants. After the near-field gain can be calculated according to (6) by use of \( S_{21} \) measured in the near-field region and fitted to the curve given by (9), the far-field gain \( G_{\text{dB}} \) in dB representation can be determined.
Table 1: Setting parameters of Network Analyzer, Agilent N5230A

<table>
<thead>
<tr>
<th>Setting item</th>
<th>Before inserting amplifier</th>
<th>After inserting amplifier</th>
</tr>
</thead>
<tbody>
<tr>
<td>IF Band Width [Hz]</td>
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<td>30</td>
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<tr>
<td>Averaging</td>
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<td>Sweep Type</td>
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<tr>
<td>Number of point</td>
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<td>−15</td>
</tr>
<tr>
<td>Source Attenuator [dB]</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

![Graphs](image)

(a) before including the amplifier  
(b) after including the amplifier

Figure 2: Estimated far-field gain by curve fitting for $S_{21}$

3. Measurement Results of Far-Field Gain

3.1 Measurement Setup

The measured complex relative permittivity of the tissue-equivalent liquid was $39.0 - j13.4$ at 2.45GHz by the dielectric probe (Agilent 85070E). The temperature in the liquid was 21.2°C. The attenuation and phase constants are calculated as $\alpha = 473$dB/m and $\beta = 325$rad/m. For both the measurement system before and after inserting the amplifier, the distance between the two antennas, $r$, is controlled by the sliding stage and moved from 0mm to 250mm at an interval of 1mm to measure $S_{21}$ between the two antennas. $S_{11}$ and $S_{22}$ were measured at the maximum distance, $r = 250$mm. Some parameters of NA are listed in Table 1. Two offset dipole antennas with a length of 13mm are used as the reference antennas.

3.2 Far-Field Gain Estimated by Curve Fitting for $S_{21}$

Fig. 2 shows curves of estimated far-field gain as a function of $r_{stop}$ in the cases before and after inserting the amplifier. Because the far-field boundary $r_f$ is about 30mm, $r_{start}$ is fixed to be 40mm. Also, Fig. 2 includes curves of the far-field gain estimated by using $S_{21}$ measured only in the far-field region, that is, setting $r_{start} = 80$mm. For $r_{start} = 40$mm, the far-field gain converges with about $-2$dB and fluctuates over $r_{stop} = 130$mm due to the influence of the noise floor of the system before inserting the amplifier. For $r_{start} = 80$mm, the far-field gain is unstable due to the influence of the noise floor. For example, the value of the far-field gain is $-1.98$dB for the fitting range of 40mm–100mm. For the system after inserting the amplifier, the far-field gain well converges with about $-2$dB with no fluctuation until $r_{stop} = 160$mm in both cases of $r_{start} = 40$mm and 80mm. For example, the values of the far-field gain are $-1.91$dB and $-1.79$dB for the fitting range of 40mm–100mm and 80mm–130mm, respectively.
3.3 Far-Field Gain Estimated by Curve Fitting for Near-Field Gain

Fig. 3 shows the far-field gain as a function of the upper limit of the fitting range $r_{\text{stop}}$ for fixed lower limit in the cases before and after including the amplifier. By making a comparison between two figures, the far-field gain is found to converge with about $-1.8\,\text{dBi}$ and fluctuate over $r_{\text{stop}} = 110\,\text{mm}$ and $160\,\text{mm}$ for the system before and after including the amplifier, respectively. Moreover, the estimated gain is unstable when $r_{\text{stop}}$ is close to $r_{\text{start}}$ due to the nature of the asymptotic series of (9). Thus, the fitting range should be selected in consideration of the nature of the asymptotic series.

4. Conclusion

To validate the gain estimation based on curve fitting for the near-field gain derived from the extended Friis transmission formula which is valid in the near-field region, we enhance the dynamic range of the measurement system to insert the amplifier into the vector network analyzer and enable $S_{21}$ measurement in the far-field region. By making a comparison between the estimated gain based on the ordinary Friis transmission formula in the far-field region and that based on the extended Friis transmission formula in the near-field region, they give close agreement with each other. This fact leads to experimental validity of the extended Friis transmission formula that we proposed.

References

[1] IEC International Standard 62209-1, “Human exposure to radio frequency fields from hand-held and body-mounted wireless communication devices — Human models, instrumentation, and procedures — Part 1: Procedure to determine the specific absorption rate (SAR) for hand-held devices used in close proximity to the ear (frequency range of 300MHz to 3GHz),” Feb. 2005.


