High-Frequency Solution for Ground Wave Propagation along Land-to-Sea Mixed-Path

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1. Introduction

When the surface impedance of the earth along the propagation direction changes abruptly, the ground wave is strongly affected as it occurs when the radio wave traverses a coastline from the land to the sea [1]-[8]. In this study, we shall obtain the integral representation for the ground wave propagation over the land-to-sea mixed-path by applying the aperture field method. Then by applying the high-frequency analysis method in evaluating the integral, we will obtain the asymptotic solution for the land-to-sea mixed-path ground wave propagation.

In order to verify the validity of the high-frequency asymptotic solution, we will compare the asymptotic solution both with the mixed-path theory [2], [3], [5] – [7] and the experimental results [3], [5] – [10]. By examining the asymptotic solution in detail, we will find out the cause of the recovery effect [2] occurring on the sea over the land-to-sea mixed-path propagation.

2. High-Frequency Solution for Ground Wave Propagation

2.1 Formulation and Integral Representation

Fig. 1 shows the coordinate system \((x, y, z)\), the mixed-path along which the surface impedance changes at the coastline \(x = d\) from \(Z_a\) to \(Z_b\), and the aperture plane defined by \(x = d, -\infty < y < \infty, 0 \leq z < \infty\). Here, the complex dielectric constants \(\varepsilon_a\) and \(\varepsilon_b\) are defined by \(\varepsilon_a = \varepsilon_{ar} + i\sigma_a / \omega\) (\(\varepsilon_a = \varepsilon_{ar}\varepsilon_0\)) and \(\varepsilon_b = \varepsilon_{br} + i\sigma_b / \omega\) (\(\varepsilon_b = \varepsilon_{br}\varepsilon_0\)) where \(\varepsilon_{ar} (\varepsilon_{br})\) denotes the relative dielectric constant of the land (sea).

The electromagnetic field radiated from the transmitting antenna T(0, 0, 0) and received by the receiving antenna R\((x_0, 0, 0)\) can be represented by the integral form by applying the aperture field method which uses the equivalent current \(J_z\) \((d, y', z')\) induced on the aperture plane (see Fig. 1). The integral is given by the double integral with respect to \(y'\) and \(z'\) on the aperture plane. However, the integration with respect to \(y'\) can be performed by applying the isolated saddle point technique [11] for the sufficiently large \(k_0x\), i.e., for \(k_0x \gg 1\), where \(k_0\) denotes the wavenumber in the air. Thus the double integral discussed above may be reduced to the following single integral:

\[
E_{z,m} = -\frac{\omega^2 \mu_0 I}{8\pi^2 Z_0} \sqrt{\frac{2\pi}{k_0}} e^{ik_0\pi/4} I(k_0), \quad I(k_0) = \int_0^\infty f(z') e^{ik_0(z'-z')} dz'
\]  

(1)
Where the attenuation functions \( W_a(z') \) of the land and \( W_b(z') \) of the sea are given by [12] – [14]

\[
f(z') = \frac{W_a(z')W_b(z')}{\left\{ R_a(z')R_b(z')q(z') \right\}^{1/2} \sin \theta'_a}
\]

\[
q(z') = R_a(z') + R_b(z'), \quad R_a(z') = \sqrt{d^2 + z'^2}, \quad R_b(z') = \sqrt{(x-d)^2 + z'^2}
\]

Where the attenuation functions \( W_a(z') \) of the land and \( W_b(z') \) of the sea are given by [12] – [14]

\[
W_{a,b}(z') = (1 + \Gamma_{a,b}\sin^2 \theta'_{a,b}) + (1 - \Gamma_{a,b})F(p_{a,b}(z'))
\]

\[
\Gamma_{a,b} = k_{a,b}z' - k_{0}R_{a,b}(z'), \quad k_{a,b} = \alpha \sqrt{e_{a,b}^{-1}}
\]

In (4), \( F(p_{a,b}(z')) \) is the Sommerfeld’s attenuation function of the land (sea) and is defined by [2], [3], [5] – [8], [12] – [14]

\[
F(p_{a,b}(z')) = 1 + \pi \sqrt{p_{a,b}(z')}e^{-p_{a,b}(z')} \text{erfc}(-ip_{a,b}(z')).
\]

The notations \( R_a(z'), R_b(z'), \theta'_a, \) and \( \theta'_b, \) and the integration point \( P(x = d, z = z') \) located on the aperture plane used in the above equations are shown in Fig. 2.

### 2.2 High-Frequency Asymptotic Solution

The integral \( I(k_0) \) in (1) may be evaluated asymptotically by applying the saddle point technique applicable uniformly as the saddle point \( z' = z'_s \) approaches the lower limit (end point) of the integral. Following the general procedure given in [11], one may obtain the following solution:

\[
I(k_0) \approx I_1(k_0) + I_2(k_0), \quad I_1(k_0) = f(z'_s)h_1(z'_s)\sqrt{k_0}e^{i0g(z'_s)}Q(\sqrt{k_0}S_a e^{-i\pi/4})
\]

\[
h_1(z'_s) = \frac{2}{q(z'_s)}e^{i\pi/4}, \quad S_a = \sqrt{q(z'_s) - q(0)e^{i\pi/4}}, \quad Q(z) = \int_z^\infty e^{-x^2} dx, \quad Q(0) = \frac{\sqrt{\pi}}{2}
\]

\[
I_2(k_0) = f(0)h_2 - f(z'_s)h_1(z'_s)\sqrt{k_0}S_a e^{i0g(z'_s) - k_0S_a^2}, \quad h_2 = \frac{4IS_a}{q(0)}
\]

In deriving above solutions, we have utilized the results that the saddle point \( z'_s \) of the integrand in (1) in the complex \( z' \)-plane obtained from \((d/dz')q(z') = 0\) is located at the \( z' = z'_s \) and \( q(z'_s) \) is given by

\[
q(z'_s) = 0 = x/|d(x-d)| > 0.
\]

If the simplification of \( I_1(k_0) \) in (7) and \( I_2(k_0) \) in (9) is performed by applying the high-frequency approximations, one may obtain the following electric field representation for the land-to-sea mixed-path ground wave propagation.

\[
E_{z,m} = E_{z,m}^{(1)} + E_{z,m}^{(2)}
\]

Where, \( E_{z,m}^{(1)} \) denotes the electric field component of the ground wave propagating from the land to the sea traversing the coastline located at \( x = d \). The solution will be represented by
While, the second term $E^{(2)}_{z,m}$ represents the scattered electric field excited at the coastline $x = d$ by the incident ground wave $E_{in}$ in (12) and reaches the receiving antenna $R$ located on the sea (see Fig. 2). The scattered field solution may be represented by the following form:

$$E^{(2)}_{z,m} = E_{in} D(x, d, x, Z_a) E^d$$

where the incident ground wave $E_{in}$ is defined in (12) and the scattering coefficient $D$ and the scattered wave $E^d$ propagating from the coastline to the receiving antenna $R$ are defined by

$$D(x, d, x, Z_a) = 2\sqrt{2\pi k_0} \left( \frac{d(x-d)}{x} \right)^{3/2} \frac{Z_a e^{-Z_a/4}}{Z_0}$$

$$E^d = \frac{e^{ik_0(x-d)}}{2\pi(x-d)} F(p_b(0)), \quad p_b(0) = \frac{k_0(x-d)}{2} \left( \frac{Z_a}{Z_0} \right)^2$$

3. Comparisons with Mixed-Path Theory and Experimental Results

In Fig. 3, we have shown the electric field magnitude curves as the function of the distance $x$ (km). The propagation paths change from the land ($\epsilon_a = 10$, $\sigma_a = 0.001$ S/m) to the sea ($\epsilon_b = 80$, $\sigma_b = 5$ S/m) at $d = 32.5$ km (Fig. 3(a)) and $d = 38.5$ km (Fig. 3(b)). The frequencies $f = 1422$ kHz and $f = 3925$ kHz for the commercial radio broadcast are used in calculating Fig. 3(a) and Fig. 3(b), respectively. The solid curves (---) are calculated by using the asymptotic solution in (11), while the closed circles (●●●) are obtained from the conventional mixed-path theory [2], [3], [5] – [7].

It is clarified that the asymptotic solution (---) agrees excellently with the mixed-path theory (●●●). It is observed that the field magnitude is enhanced on the side of the sea over the land-to-sea mixed-path and approaches gradually the electric field amplitude of the all-sea path propagation shown by chain-dot curves (⋯⋯). Thus, the phenomenon called as the recovery effect [2], [6], [9], [10] is also realized by using the asymptotic solution derived in (11).

We have performed the experiments to compare the asymptotic solutions with the measurements. The routes 1 and 2 for the experiments are shown in Fig. 4. The experimental results along the route 1 ($f = 1422$ kHz) and the route 2 ($f = 3925$ kHz) are shown by the crosses (×××) in Fig. 3(a) and Fig. 3(b). It is clear from the results that the theoretical results obtained from the asymptotic solution in (11) and the mixed-path theory agree very well with the experimental results. The recovery effects are observed also through the experiments.

In Fig. 5(a), in order to find out the cause of the recovery effect, we have calculated the scattering coefficient $D$ in (14) and the scattered wave $E^d$ in (15). $E^d$ are calculated by using (1): ($\epsilon_{br} = 80$, $\sigma_b = 5$ S/m) corresponding to the sea, (2): ($\epsilon_{br} = 10$, $\sigma_b = 0.001$ S/m) corresponding to the land, and (3): ($\epsilon_{br} = 40$, $\sigma_b = 0.01$ S/m). The $\epsilon_{br}$ and $\sigma_b$ in (3) are some values between the sea and the land. It is observed that the amount of the increase of $D$, as the function of the distance ($x - d$) from the coastline, is greater than that of the decrease of $E^d$ with (1). On the other hand, the amount of the decreases of $E^d$ with (2) and (3) are greater than that of the increase of $D$. Therefore, the scattered field $E^{(2)}_{z,m}$ in (13) constructed by using $DE^d$ (i.e., the multiplication of $D$ and $E^d$) increases as the function ($x - d$) for the case of (1)(sea) as shown in Fig. 5(b). However, the scattered fields

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Figure 4: Routes for experiments in Kanto area in Japan.
From this study it becomes clear that the recovery effect is produced by the combination of the scattering coefficient $D$ and the scattered wave $E^d$. When the conductivity $\sigma_b$ or the relative dielectric constant $\varepsilon_{br}$ is very large compared with $\sigma_a$ or $\varepsilon_{ar}$ for the land, the scattered wave $E^d$ decreases “very slowly” as the function of $(x-d)$ (see $E^d$ with 1 in Fig. 5(a)). This is the reason why the scattered field $E_{z,m}^{(2)}$ and therefore the total field $E_{z,m} = E_{z,m}^{(2)} + E_{z,m}^{(3)}$ increase as the function of the distance $(x-d)$. Thus by investigating the results in Figs. 5(a) and 5(b), one may find out the cause of the recovery effect.

4. Conclusions

In the present study, we have derived the novel high frequency solution for the ground wave propagation over the land-to-sea mixed-path. By comparing both with the conventional mixed-path theory and the experimental results performed in Kanto area including the sea near Tokyo bay and Sagami bay in Japan, we have confirmed the validity of the high-frequency asymptotic solution proposed in this paper. We have also find out the cause of the recovery effect occurring on the side of the sea over the land-to-sea mixed-path.

References