ASED-AIM Analysis of EM Scattering by 3D
Huge-Scale Finite Periodic Arrays

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Abstract
In this paper, the Adaptive Integral Method (AIM) has been successfully extended to characterizing electromagnetic scattering by large scale finite periodic arrays with each cell comprising of either dielectric or metallic objects, by utilizing accurate sub-entire-domain (ASED) basis function. The complexity analysis shows that the computational time of $O(N_0 \log N_0) + O(M \log M)$ and memory requirement of $O(N_0) + O(M)$ in the ASED-AIM where $N_0$ denotes the number of cells and $M$ stands for the number of elements in one cell are both considerably reduced as compared to those in the existing AIM. In addition, an example with over 10 million unknowns is given to demonstrate the efficiency of the proposed method.

1. Introduction
Large-scale finite periodic structures such as metamaterials [1] have been topics of considerable interests and various novel industrial, academic and scientific applications. Therefore, accurate full wave analysis of these structures using numerical methods such as MoM are very important. When using MoM to solve these problems, the memory requirement and computational complexity are $O(N^2)$ where $N$ is number of unknowns. Recently developed fast solvers such as AIM [2]–[4] can alleviate the stringent requirements and reduce memory requirement to $O(N)$ and computational complexity to $O(N \log N)$ respectively. However, full wave simulations for these structures, even with the aid of the available fast solvers, are very difficult since important properties (such as periodicity) are not used in conventional solvers. Recently, some novel physics-based basis function have been proposed to solve these challenging problems such as accurate sub-entire-domain (ASED) basis function [5]. Using ASED basis function, the unit cell can be represented with one basis function; thus, the total number of unknowns can be brought down from $MN_0$ to $N_0$ where $N_0$ is number of cells in the whole domain and $M$ is number of unknowns in one unit cell. Therefore, ASED-AIM, which is the combination of ASED with AIM, can reduce the memory requirement from $O(MN_0)$ (using conventional AIM) to $O(M) + O(N_0)$ and reduce the computational complexity from $O(MN_0 \log MN_0)$ (using conventional AIM) to $O(M \log M) + O(N_0 \log N_0)$. Numerical examples have demonstrated the accuracy and efficiency of the proposed ASED-AIM in solving huge-scale finite periodic array problems. An example with over 10 million unknowns concludes the paper.

2. ASED-AIM Formulation
Electromagnetic scattering by periodic composite conducting and dielectric objects can be characterized using volume-surface integral equation (VSIE):

$$E'(r) = E(r) - E^s(r), \quad r \in V; \quad \text{and} \quad E'(r) \bigg|_{\tan} = -E^s(r)_{\tan}, \quad r \in S.$$  \hspace{1cm} (1)

Equivalent electric volume and surface current $J_V(r)$ and $J_S(r)$ are related to total electric field $E(r)$ and scattered electric field $E^s(r)$ via

$$J_V(r) = j\omega \kappa D(r) = j\omega(\epsilon - \epsilon_0)E(r), \quad r \in V$$  \hspace{1cm} (2)
\[ E^s(r) = -j\omega\mu_0 \int_V G(r, r')J_V(r')dV' - j\omega\mu_0 \int_S G(r, r')J_S(r')dS' \]
\[ + \frac{\nabla}{j\omega\epsilon_0} \int_V G(r, r')\nabla' \cdot J_V(r')dV' + \frac{\nabla}{j\omega\epsilon_0} \int_S G(r, r')\nabla' \cdot J_S(r')dS' \]  

(3)

where \( G(r, r') \) denotes free space Green’s function, \( \mu_0 \) and \( \epsilon_0 \) represent free space permeability and permittivity respectively, \( \epsilon \) stands for permittivity in the dielectric object, and \( \kappa = (\epsilon - \epsilon_0)/\epsilon \) identifies the contrast ratio of scatterer material and its background medium. For the \( p \)-th cell, surface currents and volume currents can be expanded as follows:

\[ J^S_p = \sum_{m=1}^{N_S} I^S_{pm} f^S_{pm}, \quad \text{and} \quad J^V_p = \sum_{m=1}^{N_V} I^V_{pm} \kappa f^V_{pm}, \]  

(4)

where \( f^S_{pm} \) and \( f^V_{pm} \) denote respectively the RWG and SWG basis functions associated with the \( m \)-th surface and volume basis functions of the \( p \)-th cell, \( N_S \) is the number of RWG basis functions while \( N_V \) is the number of SWG basis functions, and \( I^S_{pm} \) and \( I^V_{pm} \) stand for the respective unknown coefficients to be solved for. Thus, the current density can be written in terms of electric current for the \( p \)-th cell as follows:

\[ J = \sum_{p=1}^{N_0} j_p J_p \quad \text{where} \quad J_p = J^S_p + J^V_p; \]  

(5)

where \( j_p \) denotes unknowns to be solved for. After solving a small array problem, we can construct ASED basis functions for each cell and then use them to solve the entire problem. The cell impedance matrix elements (denoted by the superscript \( C \) herein and subsequently) can be written as

\[ Z^C_{pq} = \sum_{m=1}^{M} \sum_{n=1}^{M} I_{pm} Z_{pm, qn} I_{qn}. \]  

(6)

Using conventional AIM, the matrix vector multiplication can be written as

\[ ZI = \nabla H P I + Z_{\text{near}} I \]  

(7)

where \( \nabla \) is the interpolation matrix, \( H \) is Green’s function matrix, and \( P \) is the projection matrix. The four steps of conventional AIM can be shown in Fig. 1(a). For the far zone interaction, the impedance matrix elements can be approximated as:

\[ Z_{pm, qn} \approx \tilde{Z}_{pm, qn} = \sum_s \sum_t V_{ms} H_{m_s n_t} P_{nt}. \]  

(8)

Figure 1: The pictorial representation of (a) conventional AIM and (b) ASED-AIM. Step 1 denotes the projection; Step 2 stands for the grid potential calculation using FFT; Step 3 represents the interpolation; and Step 4 identifies the direct calculation of near zone interactions. Shaded area denotes the near zone.
where \( \sum \) denotes summation of all the grids associated with the basis functions. Thus, for cell interaction in the far zone, we have

\[
Z_{pq} = \sum_{m} \sum_{n} I_{pm} Z_{pm,qn} I_{qn} \approx \sum_{m} \sum_{s} \sum_{n} I_{pm} V_{m,s} H_{m,s,n} I_{qn} P_{ni} = V_{p}^{C} H_{pq} P_{q}^{C}
\]

(9)

where \( V_{C} \) and \( P_{C} \) are the interpolation and projection matrices for cell basis functions. They can be written explicitly as:

\[
V_{p}^{C} = \sum_{m} \sum_{s} I_{pm} V_{m,s}, \quad \text{and} \quad P_{q}^{C} = \sum_{n} \sum_{t} I_{qn} P_{n,t}.
\]

(10)

Now, Using ASED-AIM, the matrix vector multiplication can be written as

\[
Z^{C} \cdot I_{C} = V_{C} \cdot H \cdot P_{C} \cdot I_{C} + Z^{C,\text{near}} \cdot I_{C}
\]

(11)

The four steps for implementing ASED-AIM is shown graphically in Fig. 1(b).

3. Numerical Results

In this section, several examples will be given to demonstrate the validity and efficiency of our code to solve electromagnetic scattering by large scale periodic structures consisting of composite metallic and dielectric objects. In all the examples, the periodicity of the arrays in \( x \)-, \( y \)-, and \( z \)-directions are all \( 0.2 \lambda \). The arrays are under normal incidence at \( \theta_{i} = 0^\circ \) and \( \phi_{i} = 0^\circ \) with electric field theta-polarized. First, we consider a 2D periodic structure shown in

![Figure 2: Examples of arrays used in the calculations of numerical results. (a) The structure of a unit cell, \( d = 0.2 \lambda_{0} \). The shaded area denotes a metallic patch while cube is a dielectric object with \( \epsilon = 2.2 \). (b) \( 4 \times 4 \) array. (c) \( 4 \times 4 \times 4 \) array. (d) \( 100 \times 100 \) array.](image)

Fig. 2(b), which is a \( 4 \times 4 \) array in \( xy \) plane. The results shown in Fig. 3(a) are generated by ASED-AIM and conventional AIM. Excellent agreement has been observed. Subsequently, we investigate the computational complexity and memory requirement of ASED-AIM and compare them with those of the conventional AIM. Figs. 4(a) and (b) respectively show the relationship between the computational time and memory requirement with the number of unknowns using ASED-AIM and AIM. From these figures, it is clear that ASED-AIM is much more efficient in solving periodic array problems than AIM. The above method can be also extended to analyze 3D finite periodic structures. In Fig. 3(b), the RCS values calculated using ASED-AIM and AIM are compared for the \( 4 \times 4 \times 4 \) array shown in Fig. 2(c) and good agreement have been observed. Finally, we consider an electrically very large finite periodic structure with \( 100 \times 100 \) array shown in Fig. 2(d). The total number of unknowns in this example is 10.87 million. The calculated radar cross section is shown in Fig. 3(c). For such an electrically large structure with over 10 million unknowns, ASED-AIM only requires 273 MB memory and 1200 seconds, which demonstrates the efficiency of the new method in solving problems of electromagnetic scattering by large-scale periodic structures.
Figure 3: Bistatic RCS of the (a) $4 \times 4$ array, (b) $4 \times 4 \times 4$ array, and (c) $100 \times 100$ array (leading to over 10 million unknowns); with each cell shown in Fig. 2(a).

Figure 4: The relationship between (a) computational time (b) memory requirement and the number of unknowns using ASED-AIM (triangle line) and AIM (circle line).

4. Conclusions

In this paper, a new algorithm integrating the ASED basis functions into the AIM has been developed to solve problems of scattering by huge-scale finite periodic arrays comprising of metallic and dielectric objects. It has thus reduced the memory requirement and computational time significantly in solving the array problems. High accuracy and efficiency of the ASED-AIM has been demonstrated. An example with over 10 million unknowns is successfully considered in a personal computer and its numerical results are illustrated.

References