A New Aspect on Wedge Diffraction: Hidden Rays on the Shadow Boundary

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1. Introduction

The phenomenon of diffraction may be defined by any deviation of geometrical rays from rectilinear paths which cannot be interpreted as reflection and refraction. Although the ascending series solutions to some canonical structures as cylinders, spheres, conducting wedges, etc. are available, they are not directly useful in high frequency region. Those ascending series solutions suffer from very slow convergence in short wavelength region and no way out for explaining the diffraction phenomenon. For the perfectly conducting half-plane illuminated by a plane wave, Sommerfeld [1] provided an asymptotic solution, of which the leading and second terms agreed with the geometrical optics and edge-diffracted field, respectively. The Sommerfeld-type diffraction coefficients of perfectly conducting wedges led Keller [2] to formulate the geometrical theory of diffraction (GTD), and later many researchers to develop its improved versions. However, the application of the GTD and its improved versions to penetrable scatterers has been hampered by the lack of exact diffraction coefficients of penetrable wedges.

A number of analytical techniques have been investigated to obtain diffraction coefficients of penetrable wedges. But there is no rigorous solution to those diffraction problems until now. One of severe difficulties posed in these problems is that two propagating waves inside and outside a penetrable wedge make it impossible to satisfy the boundary condition on the wedge interfaces. While accurate values of the diffraction coefficients have been obtained using the method of moment and FDTD method, those numerical results could not provide comparable achievements in the physical understanding of the wedge diffraction. As a detour, the simplest way to obtain an approximate but analytic expression on the diffraction coefficients of penetrable wedges may be the physical optics (PO) approximation. But the PO diffraction coefficients cannot include the reflection and refraction on the shadow boundary of wedges. Hence one of the historically tantalizing problems in electromagnetic and optical area may be how to account the field behavior on the shadow boundary of penetrable wedges.

Recently the concept of hidden rays was suggested to account the effect of the shadow boundary of penetrable wedges using the same analytic process as the ordinary rays on the lit boundary. This approach was called the hidden rays of diffraction (HRD), which have already been used to construct analytic expressions of the diffraction coefficients of two penetrable wedges. One is a composite wedge composed of a perfect conductor and a lossless dielectric [3], and the other is a lossless dielectric wedge [4]. The accuracy of the HRD diffraction coefficients was assured in comparison with the conventional PO diffraction coefficients.

In this paper, E-polarized diffraction by a perfectly conducting wedge is reconsidered to show the relationship between the geometrical rays and the diffraction coefficients. Hidden rays are also geometrical rays satisfying the usual principle of geometrical optics (GO). While ordinary rays exist in the physical region, hidden rays occur only in the complementary region, in which the original media inside and outside wedges are exchanged each other [5]. And the rule of hidden-ray tracing in the complementary region can be found easily. Then the HRD diffraction coefficients are constructed by sum of cotangent functions, which correspond to the geometrical rays on by one. This HRD approach is applied to the diffraction by a composite wedge composed of a perfect conductor and lossy dielectric, of which HRD diffraction coefficients are plotted here.
2. Perfectly Conducting Wedge

Consider a unit plane wave incident only on the \( \theta = \theta_1 \) boundary of the perfectly conducting wedge. Ordinary ray-tracing is shown in Fig. 1(a). In this case, the exact diffraction coefficients are well known as sum of four cotangent functions in the last column of Table 1. But the PO diffraction coefficients consist of two cotangent functions which have one-to-one correspondence to two ordinary rays, as shown in the third column of Table 1. The PO diffraction coefficients become exactly equal to two terms among the exact diffraction coefficients if those angular periods are changed from \( 2\pi \) to \( 2\pi \nu_\infty \). The value \( \nu_\infty \) is obtained from the edge condition at the wedge tip. In Table 1, two additional cotangent functions remain in the exact diffraction coefficients. Then, one may guess that two remaining cotangent functions are also generated from two geometrical rays. This reverse procedure is illustrated in the last two rows in Table 1. Then, one may obtain two geometrical rays, which are called hidden rays.

It is well known that the asymptotic integration of the ordinary PO diffraction coefficients provides not only the GO term but also the edge-diffracted field in the physical region. In contrast, the asymptotic integration of the hidden PO diffraction coefficients contributes to only the edge-diffracted field in the physical region because the hidden rays cannot appear in the physical region. Then, the problem is how to estimate the amplitudes and propagation angles of hidden rays. Fig. 1(b) shows the trajectory of two hidden rays, which correspond to the incident and reflected rays on the shadow boundary of the perfectly conducting wedge. Introducing the supplementary screen, as shown in Fig. 1(b), renders the hidden rays in the complementary region to be visible.

Table 1: Relationship between Geometrical Rays and Diffraction Coefficients of a Perfectly Conducting Wedge

<table>
<thead>
<tr>
<th>Geometrical Rays</th>
<th>Ordinary PO</th>
<th>Hidden PO</th>
<th>Exact</th>
</tr>
</thead>
<tbody>
<tr>
<td>( e^{j\bar{k_0} \rho \cos(\theta_1 - \theta_\perp)} )</td>
<td>(- \frac{1}{2} \cot \left( \frac{w - \theta_\perp}{2} \right) )</td>
<td>(- \frac{1}{2} \frac{1}{\nu_\infty} \cot \left( \frac{w - \theta_\perp}{2 \nu_\infty} \right) )</td>
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<tr>
<td>( R_\infty e^{j\bar{k_0} \rho \cos(\theta_1 - \theta_\perp)} )</td>
<td>(- \frac{1}{2} R_\infty \cot \left( \frac{w - \theta_\perp}{2} \right) )</td>
<td>(- \frac{1}{2} \frac{1}{\nu_\infty} R_\infty \cot \left( \frac{w - \theta_\perp}{2 \nu_\infty} \right) )</td>
<td></td>
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<tr>
<td>( e^{j\bar{k_0} \rho \cos(\theta_1 - \theta_\parallel)} )</td>
<td>( \frac{1}{2} \cot \left( \frac{w - \theta_\parallel}{2} \right) )</td>
<td>( \frac{1}{2} \frac{1}{\nu_\infty} \cot \left( \frac{w - \theta_\parallel}{2 \nu_\infty} \right) )</td>
<td></td>
</tr>
<tr>
<td>( R_\parallel e^{j\bar{k_0} \rho \cos(\theta_1 - \theta_\parallel)} )</td>
<td>( \frac{1}{2} R_\parallel \cot \left( \frac{w - \theta_\parallel}{2} \right) )</td>
<td>( \frac{1}{2} \frac{1}{\nu_\infty} R_\parallel \cot \left( \frac{w - \theta_\parallel}{2 \nu_\infty} \right) )</td>
<td></td>
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</table>
It should be noted that hidden rays exist in the complementary region. The concept of the complementary region was derived from the formulation of the dual integral equations to the diffraction by a wedge [5]. In particular, the accuracy of the HRD diffraction coefficients in the physical region can be checked by showing how closely the HRD diffraction coefficients approach to zero in the complementary region.

The HRD procedure may be summarized as follow: 1) Not only ordinary rays in the physical region but also hidden rays in the complementary region are traced only by employing the usual principle of GO. 2) Assume a geometrical ray \( u_\nu(\rho, \theta) \) with propagation angle \( \theta_\nu \) and amplitude \( K_\nu \). The corresponding component of the HRD diffraction coefficients can be expressed directly by \((-1)^\tau \frac{1}{2\nu_\tau} K_\tau \cot \left( \frac{\omega - \theta_\tau}{2\nu_\tau} \right) \). 3) \( \tau \) is taken by 0 or 1 in case that the normal direction from the boundary illuminated by the geometrical ray \( u_\nu(\rho, \theta) \) to the physical region is positive or negative \( \theta_\nu \), respectively. 4) \( \nu_\epsilon \) is taken by the minimum positive value satisfying the edge condition at the wedge tip.

3. Wedge Composed of a Perfect Conductor and a Lossy Dielectric

![Figure 2: E-Polarized Diffraction by a Composite Wedge Composed of a Perfect Conductor and a Lossy Dielectric](image)

Fig. 2 shows the geometry of a composite wedge consisting of a perfect conductor in \( S_c \) and a lossy dielectric with complex relative dielectric constant \( \varepsilon' = \varepsilon_\epsilon + i\varepsilon_\eta \) in \( S_d \). Consider an E-polarized unit plane wave \( u_i(\rho, \theta) \) with an arbitrary angle \( \theta_i \) incident on the wedge. At first, ordinary rays are easily traced by employing the usual principle of GO in the physical region even if the dielectric may be lossy. The modified propagation constants for non-uniform plane wave transmission through conducting media [6] provide the GO field including the multiple reflections inside the lossy dielectric part. When the ordinary ray-tracing is terminated after multiple reflections inside the lossy dielectric wedge, the hidden rays are traced in the complementary region. According to one-to-one correspondence, the diffraction coefficients are constructed routinely from the ray-tracing data. The angular period of the diffraction coefficients is adjusted to satisfy the edge condition at the tip of the composite wedge. Then, the HRD diffraction coefficients are given by finite series of the cotangent functions.

As a typical example in Fig. 2, the composite wedge composed of lossy dielectric \( (\theta_i = 60^\circ, \varepsilon_\epsilon = 1.01 \text{ and } \varepsilon_\eta = 0.1-100) \) and perfect conductor \( (\theta_i = 330^\circ) \) is illuminated by an E-polarized unit plane wave \( (\theta_i = 170^\circ) \). The real parts of the PO and HRD diffraction coefficients in the air region are plotted in Figs. 3(a) and (b), respectively. The dotted \( (\varepsilon_\eta = 0) \) and broken \( (\varepsilon_\eta = \infty) \) lines denote the exact solutions to the perfectly conducting wedges with \( \theta_i = 0^\circ \) and...
60°, respectively. $S_y^{(0)}$ and $S_c^{(0)}$ denote the complementary air regions, in which the original media in $S_y$ and $S_c$ are replaced by the air, respectively. All of the PO curves in Fig. 3(a) intersect two exact patterns and cannot become zero on the conducting boundary. In contrast, the HRD diffraction coefficients in Fig. 3(b) approach the corresponding exact diffraction coefficients monotonically as $\varepsilon_i$ decreases to 0.1 or increases to 100. According to the formulation of dual integral equations, the exact diffraction coefficients should become zero in the complementary regions. Unlike to the PO diffraction coefficients in Fig. 3(a), the HRD diffraction coefficients in Fig. 3(b) satisfy the null-field condition [5] in the $S_y^{(0)}$ and $S_c^{(0)}$ quite well.

![Figure 3: Diffraction coefficients for $\theta_d = 60°$, $\theta_c = 330°$, $\theta_i = 170°$, $\varepsilon_i = 1.01$, and $\varepsilon_i = 0.1$ to 100 (dotted: exact for $\varepsilon_i = 1$ and $\varepsilon_i = 0$, broken: exact for $\varepsilon_i = \infty$).](image)

4. Conclusion

In case of a perfectly conducting wedge, it was illustrated clearly that hidden rays played a key role on accounting the reflection on the shadow boundary. For an E-polarized diffraction by a composite wedge consisting of perfect conductor and lossy dielectric, the HRD diffraction coefficients were constructed by employing only the ray-tracing data and the edge condition. Compared to the PO solution, the HRD diffraction coefficients provide smooth transition behavior as $\varepsilon_i$ increases from 0.1 to 100. And the accuracy of the HRD diffraction coefficients was assured by showing how closely the null-field condition is satisfied in the complementary region.

References