1. Introduction

Rough surface parameter estimation is one of the important problems in the field of radar remote sensing that retrieve geophysical parameters such as soil roughness, moisture, and dielectric constant from synthetic aperture radar data. In theoretical researches of this kind of problem, Gaussian random rough surface models that can be characterized by two roughness parameters, the root-mean-square height and correlation length, are often employed for analytical calculations. When we compare the theoretical scattering data with actual experimental data, it is often required to estimate these roughness parameters from a surface height-profile measured by scanning devices such as a laser profiler. For accurate estimation of these parameters, data samples with sufficiently long record length are necessary [1][2]. However, the criterion of the data length required for accurate estimation of these parameters is not clear. In our previous study [3][4], we have revealed a relationship between the data record length and accuracy of estimated parameters. In the study, we have assumed that a sampling interval of surface profile measurement is sufficiently small. However, accuracy of the correlation length estimate seems to be affected by the sampling interval because a surface slope that is a derivative of the height function is used for the calculation of the correlation length. In this study, we consider the influence of the sampling interval of measurement on accuracy of the correlation length estimate and check the result through a Monte Carlo simulation.

2. Rough Surface Parameter Estimates

First, we briefly summarize the results of our previous study [4]. Figure 1 shows a one-dimensional Gaussian random rough surface whose profile is described by a height function $z = f(x)$ with mean value zero and its statistical properties are invariant under the translation of spatial coordinate $x$ (stationary). Two roughness parameters, the rms-height $h$ and correlation length $l_c$ can be estimated using surface height profile as follows:

$$h^2 = \lim_{L \to \infty} \frac{1}{L} \int_{-L/2}^{L/2} f^2(x) \, dx,$$

$$l_c^2 = \frac{2h^2}{s^2}.$$  \hspace{1cm} (1)

where $s$ is the rms-slope defined by

$$s^2 = \lim_{L \to \infty} \frac{1}{L} \int_0^L (df/dx)^2 \, dx.$$  \hspace{1cm} (2)

When the data record length is finite, these parameters are estimated by averaging them over the finite length $L$ as follows:

$$\hat{h}^2 = \frac{1}{L} \int_0^L f^2(x) \, dx,$$

$$\hat{l}_c^2 = \frac{2\hat{h}^2}{\hat{s}^2}$$  \hspace{1cm} (3)

with

$$\hat{s}^2 = \frac{1}{L} \int_0^L (df/dx)^2 \, dx$$  \hspace{1cm} (4)

Fig. 1: Gaussian random rough surface
where the mark \( ^\wedge \) denotes an estimator. As the data record length \( L \) increases, accuracy of the estimates increases. In our previous study, we have statistically estimated the errors arising from the data record length based on the interval estimates and have derived an analytical expression of data record length required for given accuracy. For mean square height estimate, we define the normalized mean square error \( \varepsilon_L^2 \) with 95% confidence as follows:

\[
\varepsilon_L^2 = \frac{(1.96)^2 \text{Var}[\hat{h}]}{L} = \frac{\pi (1.96)^2 l_c}{2L} \tag{5}
\]

where \( \text{Var}[\cdot] \) means a variance. The square root of above equation gives the normalized rms-error and it decreases slowly \( (\sim L^{-1/2}) \) as the data length \( L \) increases. This means that a comparatively long data length is required to reduce the estimation error. After solving Eq. (5) with respect to \( L \), we have \( L \approx 240 l_c \) for 10% rms-error. This result indicates that, “in order to estimate the rms-height with a precision of \( \pm 10\% \), data record length \( L \) should be at least \( 240l_c \).”

For correlation length estimate, it should be noted that the uncertainties in estimates \( \hat{h}^2 \) and \( \hat{s}^2 \) affect the uncertainty in \( \hat{l}_c \), and thus an error propagation analysis should be required for error estimation of \( \hat{l}_c \). After omitting complicated but straightforward calculations, we can lead to the following relation:

\[
\text{Var}[\hat{\ell}_c] = \frac{1}{4} \left[ \frac{\text{Var}[\hat{h}^2]}{(h^2)^2} + \frac{\text{Var}[\hat{s}^2]}{(s^2)^2} - 2 \frac{\text{Cov}[\hat{h}^2\hat{s}^2]}{h^2 s^2} \right] \approx \frac{3}{8} \sqrt{\frac{\pi}{2}} \frac{l_c}{L} \tag{6}
\]

where \( \text{Cov}[\cdot] \) means a covariance. Similar to the rms-height estimate, we define the normalized mean square error \( \varepsilon_{lc}^2 \) with 95% confidence as follows:

\[
\varepsilon_{lc}^2 = \frac{(1.96)^2 \text{Var}[\hat{\ell}_c]}{l_c^2} = \frac{(1.96)^2 \cdot 0.470}{L} l_c \tag{7}
\]

After solving Eq. (7) with respect to \( L \), we have \( L \approx 181l_c \) for 10% rms-error. This result indicates that, “in order to estimate the correlation height with a precision of \( \pm 10\% \), data record length \( L \) should be at least \( 180l_c \).”

### 3. Influence of Sampling Interval on Accuracy

For the correlation length estimate, data record length required for given accuracy is shorter than that for the rms-height estimate. However, accuracy of the correlation length estimate seems to be affected by a finite sampling interval of surface profile measurement, because the surface slope \( s \) is calculated by using the derivative of the height function \( (df/dx) \). Thus, we here consider the influence of the sampling interval of measurement on accuracy of the correlation length estimate. We approximate the surface slope by a central derivative \( (df/\Delta x) \) as follows:

\[
\left. \frac{df}{dx} \right| = \frac{f(x + \Delta x) - f(x - \Delta x)}{2\Delta x} = f'(x) + \frac{f''(x)}{6}(\Delta x)^2 + O((\Delta x)^4) \tag{8}
\]

where \( \Delta x \) is the sampling interval of measurement, and then, an approximated mean square slope can be written as

\[
s_a^2 = \left\langle \left( \frac{df}{dx} \right)^2 \right\rangle = \left\langle (f'(x))^2 \right\rangle + \frac{\left\langle f'(x)f''(x) \right\rangle}{3}(\Delta x)^2 + O((\Delta x)^4) \approx s^2 + \frac{\left\langle f'(x)f''(x) \right\rangle}{3}(\Delta x)^2 \tag{9}
\]

where \( \left\langle \cdot \right\rangle \) indicates ensemble averaging operation. We can find from Eq. (9) that the effect of the finite sampling interval on the mean slope estimate is order of \( (\Delta x)^2 \). Substitute Eq. (9) into Eq. (3) yields
After calculation of the term $\langle f'(x) f''(x) \rangle$ in the right hand side, we can obtain the following final result for the correlation length estimate with the finite interval $\Delta x$ as

$$l_{c\Delta} \approx l_c \left[ 1 + \left( \frac{\Delta x}{l_c} \right)^2 \right]$$

(11)

This means that the error arising from sampling interval is order of $(\Delta x)^2$ with plus sign, and thus, Eq. (3) always overestimates the correlation length. However, the error is not serious because it is order of $(\Delta x)^2$ and becomes less than 1% when the sampling interval is less than a tenth of true correlation length $(\Delta x \leq 0.1 l_c)$. Furthermore, if we solve Eq. (11) with respect to $l_c$, we have

$$l_c \approx l_{c\Delta} \left[ 1 - \left( \frac{\Delta x}{l_{c\Delta}} \right)^2 \right]$$

(12)

The right hand side of above equation includes $l_{c\Delta}$ and $\Delta x$ only, and thus, we can remove the error from the estimate $l_{c\Delta}$.

3. Monte Carlo Simulation

In order to check the result obtained above, a Monte Carlo simulation is carried out. For sample data generation, we employ the well-known spectral method and make 400 rough surface realizations with roughness parameters $h = l_c = 1.0 \text{ cm}$. Figure 2 shows correlation length estimates and corresponding 95% confidence limits as a function of $L$ for $(\Delta x/l_c)^2 = 0.01$ and 0.1. In Fig. 2(a) and (c), 400 plots of the estimates calculated by Eq. (3) are shown, and in (b) and (d), 95% confidence limits of them are shown together with lines of $\pm 10\%$ margin of error (0.9 cm and 1.1 cm). We can find from Fig. 2(b) that the 95% confidence limits come into $\pm 10\%$ margin of error when the data lengths are around $180 l_c$. Furthermore, we can also find from Fig. 2(d) that the correlation length estimates for $(\Delta x/l_{c\Delta})^2 = 0.1$ are overestimated about 10%. These results agree quite well with our theoretical expectation given above, and thus, the validity of our theoretical results is confirmed.

4. Conclusions

In this study, we have estimated the error of correlation length of Gaussian rough surfaces arising from the sampling interval of surface profile measurement and have theoretically revealed the error property. The results of a Monte Carlo simulation have agreed quite well with the theoretical expectation.

References

Fig. 2. The correlation length estimates and 95% confidence limits as a function of the data length $L$ for 400 surface realizations with the roughness parameters $h = l_c = 1.0\text{cm}$. In (a) and (c), 400 plots of the estimates calculated by Eq. (3) for $(\Delta x/l_c)^2=0.01$ and 0.1 are shown, respectively. In (b) and (d), 95% confidence limits of them together with lines of $\pm 10\%$ margin of error (0.9 cm and 1.1 cm).