Basic characteristics of anisotropic artificial dielectric resonator

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1. Introduction

Artificial dielectrics are fabricated by small metal particles distributed in the host material such as plastics, rubber or ceramics. Free electrons in the metal particles are polarized by the applied electric field to make the effective permittivity quite large. Besides that, it has additional unique features, diamagnetism, big anisotropy, controllable non-uniformity and so on [1].

Anisotropy is the topic we study here. There are two mechanisms that realize and control the anisotropy of artificial materials, shape of the constituting particles (unit particles) and their arrangement. The ratio of permittivity could be more than 1000 according to the directions if one chooses a thin and long metal strip as a unit particle, for example.

The basic properties of a resonator are resonant frequency, unloaded Q and electromagnetic fields. Though anisotropy makes it complicated and difficult to solve the problem, we can still manipulate Maxwell equations analytically, as long as we restrict ourselves in a rather symmetric structure. This way, we could be able to have a physical insight into the electromagnetic properties of artificial dielectric resonators without relying on the time-consuming E/M wave simulations.

2. Permittivity and permeability of artificial dielectrics

Effective permittivity and permeability can be calculated numerically with E/M simulation software like HFSS. Typical structure for calculation is shown in Fig.1, where the upper and lower surfaces are assumed perfectly conducting, while the side surfaces magnetically conducting. It makes the transversely periodic structure of infinite extent, and hence the reflection and transmission coefficient \( S_{11} \) and \( S_{21} \) for the incident plane wave are calculated numerically for the structure in Fig.1 (a).

Since \( S_{11} \) and \( S_{21} \) for the equivalent continuous medium in Fig.1 (b) is related with the material constants theoretically as follows

\[
z = \frac{[(1 + S_{11})^2 - S_{22}^2]}{[(1 - S_{11})^2 - S_{22}^2]} \tag{1}
\]

\[
n = \frac{1}{kd} \cos^{-1} \left[ \frac{1}{2S_{21}} \left( 1 - S_{11}^2 - S_{22}^2 \right) \right] \tag{2}
\]

\[
\varepsilon = z/n \tag{3}
\]

\[
\mu = nz \tag{4}
\]

we have calculated the equivalent permittivity and permeability substituting numerically obtained \( S_{11} \) and \( S_{21} \) into the equations above[2].

Figures 2 and 3 are the typical results that characterize the anisotropy of artificial dielectrics for a disk particle which shows a uniaxial anisotropy.
3. Analysis of anisotropic dielectric resonator in a rectangular waveguide

Though the anisotropy shown in Figs. 2 and 3 is quite simple, it is distinctively different from the isotropy, and thus, we will analyze the resonator with that anisotropy shown in Fig. 4. Maxwell equations for the E/M field components inside the resonator are

\[
\text{rot} \mathbf{E} = -j \omega \varepsilon \mathbf{H} \quad \text{rot} \mathbf{H} = (\sigma + j \omega \mu) \mathbf{E}
\]

where

\[
\varepsilon = \begin{bmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}, \quad \mu = \begin{bmatrix} \mu_x & 0 & 0 \\ 0 & \mu_y & 0 \\ 0 & 0 & \mu_z \end{bmatrix}
\]

The field components for TE modes are obtained as...
The propagation constant $\beta$ inside the resonator is obtained as

\[
E_x = \frac{A_j \omega \mu_z}{b} \frac{m \pi}{\beta} \cos \left( \frac{n \pi}{a} x \right) \sin \left( \frac{m \pi}{b} y \right) - \frac{\beta^2 + \omega^2 \varepsilon_x \mu_z}{b} \frac{n \pi}{a} \sin \left( \frac{n \pi}{a} x \right) \cos \left( \frac{m \pi}{b} y \right)
\]

\[
H_y = \frac{A_j \beta}{b} \frac{m \pi}{a} \sin \left( \frac{n \pi}{a} x \right) \cos \left( \frac{m \pi}{b} y \right)
\]

\[
H_z = \frac{A \cos \left( \frac{n \pi}{a} x \right) \cos \left( \frac{m \pi}{b} y \right)}{A_j \omega \mu_z} \frac{m \pi}{b} \cos \left( \frac{n \pi}{a} x \right) \sin \left( \frac{m \pi}{b} y \right)
\]

\[
E_z = 0
\]

The propagation constant $\beta$ inside the resonator is obtained as

\[
\beta^2 = \frac{\mu_z}{\varepsilon_x} \left( \frac{n \pi}{a} \right)^2 + \frac{\mu_x}{\varepsilon_y} \left( \frac{m \pi}{b} \right)^2
\]  

(8.a)

whereas the field is assumed to decrease exponentially outside of the resonator, the rate being given

\[
\alpha^2 = \left( \frac{n \pi}{a} \right)^2 + \left( \frac{m \pi}{b} \right)^2 - \omega^2 \varepsilon_0 \mu_0
\]  

(8.b)

Considering the symmetry of the resonator, the resonant modes are divided into the even or odd modes, which are given by assuming the PMC (perfect magnetic conductor) and PEC (perfect electric conductor) at the center of the resonator as shown in Fig.5, respectively. The corresponding resonant conditions are

\[
\cot \left( \frac{\beta d}{2} - s \pi \right) = \frac{\beta \mu_0}{\alpha \mu_z} \quad \text{(even mode)}
\]

(9.a)

\[
\tan \left( \frac{\beta d}{2} - s \pi \right) = -\frac{\beta \mu_0}{\alpha \mu_z} \quad \text{(odd mode)}
\]

(9.b)

The TM resonant modes are analyzed in the same way, which provides the dual expressions for all quantities in eqs.(7) through (9).

4. Resonant frequency

The resonant frequency is calculated by solving eq.(9) with the help of eq.(8). The result is shown in Fig.6 whose data is taken from typical result for disk particles in Fig.3 (b). The resonator dimensions are 40\,*\,20\,*\,4\,mm, and it is contacted with the walls of waveguide WRJ-6. It is seen that the frequency drops down as the permittivity increases.

By substituting the imaginary part in addition to the real part into eqs.(8) and (9), one can also calculate the unloaded $Q$ of the resonator. The result is shown in Fig.7 together with the numerical result by HFSS for $\varepsilon_x=10.0$, $\varepsilon_z=2.2$, $\mu_x=1.0$, $\mu_z=0.5$, $\tan\delta_{\varepsilon_x}=0.004$, $\tan\delta_{\varepsilon_z}=0.008$, $\tan\delta_{\mu_x}=0.006$ and $\tan\delta_{\mu_z}=0.018$.  

![Mode chart](image1)

![Unloaded Q for thickness of resonator](image2)
5. Electromagnetic field distribution
Large permittivity and its anisotropy may change the electromagnetic field distribution from the conventional dielectric resonator. Figure 8 shows it along the plane y=10.0mm. The field distribution along z-axis is quite unique compared with the isotropic case shown in Fig.9.

![Fig.8 2-dimensional electromagnetic field distribution for anisotropic resonator (a=40mm, b=20mm, d=10mm, ε_tr=40.0, ε_zr=2.2, µ_tr=1.0, µ_zr=0.12)](image)

![Fig.9 2-dimensional electromagnetic field distribution for isotropic resonator (a=40mm, b=20mm, d=10, ε_tr=40.0, ε_zr=40.0, µ_tr=1.0, µ_zr=1.0)](image)

6. Conclusion
We have analyzed the effect of anisotropy induced by the artificial dielectrics to the rectangular resonator. Though the anisotropy considered here is one of the simplest cases, some unique features are found. The next step should be the experimental confirmation. More complicated anisotropy could be handled analytically later.

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References