Measurement using the S-parameter method for radiation characteristics and mutual coupling of multiport antennas on a small ground

#Takashi Yanagi, Toru Fukasawa, Hiroaki Miyashita, and Yoshihiko Konishi
Mitsubishi Electric Corporation
5-1-1 Ofuna, Kamakura, Kanagawa 247-8501, Japan
Yanagi.Takashi@bp.MitsubishiElectric.co.jp

1. Introduction

If a coaxial cable is connected to a small antenna, unbalanced current will be induced on the outside conductor of the coaxial cable. As a result, it may not be possible to measure the original antenna characteristics. In order to reduce the unbalanced current, a balun is often used. However, the operation frequency range of a balun is narrow in principle because the effect of current reduction depends on a resonance of a quarter-wavelength. Meanwhile, a measurement method using fiber optics was proposed [1]. However, it was reported that impedance is altered by the influences of the optical/electric (o/e) converter mounted on the ground plane when the size of the antenna or the ground plane is much smaller than a wavelength. The S-parameter method [2] is known to be one way to reduce the influence of an unbalanced current. Although the S-parameter method is an effective and useful method, it has only been applied for impedance measurements.

In this paper, accurate measurements using the S-parameter method, including radiation characteristics and mutual coupling between antennas mounted on a small ground plane, are proposed. First, the formulation of the proposed method is shown. Further, the effectiveness of the proposed method is experimentally confirmed.

2. Formulation for measurement by S-parameter method

2.1 Impedance and mutual coupling

Consider a case in which small \( n \)-element antennas are mounted on a small ground plane. Figure 1 shows the measurement model for the S-parameter method at \( k \)-th antenna element (\( k = 1, 2, \ldots, n \)). In this case, the \( n \)-element antenna system is expressed as \( 2n \)-port network. The incident waves at Port: \( 2k-1 \) and Port: \( 2k \) are defined as \( a_{2k-1} \) and \( a_{2k} \), and the reflected waves at Port: \( 2k-1 \) and Port: \( 2k \) are defined as \( b_{2k-1} \) and \( b_{2k} \) (\( k = 1, 2, \ldots, n \)), respectively. The incident waves, reflected waves, and S-parameters can be related as follows.

\[
SB = SA
\]

\( S \) is the scattering matrix, \( A \) is a column vector in which the element is the incident wave at each port, and \( B \) is a column vector in which the element is the reflected wave at each port.

The current on the inner conductor of the cable connected to Port: \( l \) is defined as \( i_l \). The condition in which the currents on the outside of two cables cancel each other is expressed below.

\[
i_{2k-1} = -i_{2k} \quad (k=1, 2, \ldots, n)
\]

(2)

If the characteristic impedance is normalized to 1, this condition is rewritten as follows.

\[
a_{2k-1} + a_{2k} - b_{2k-1} - b_{2k} = 0 \quad (k=1, 2, \ldots, n)
\]

(3)

On the other hand, the measurement model for a cable connected directly is shown in Fig. 2. This case corresponds to the conventional measurement. In Fig. 2, the cable is represented by a transmission line. The incident waves and the reflected waves at Port: \( K \) are defined as \( A_k \) and \( B_k \), respectively (\( k = 1, 2, \ldots, n \)). When the \( m \)-th antenna is measured, the reflection coefficient at Port: \( m \) and the transmission coefficient to the other port are obtained under the following condition.

\[
A_k = 0 \quad (k=1, 2, \ldots, n. \ k \neq m)
\]

(4)

From equation (4), we obtain

\[
a_{2k-1} + 2a_{2k} - b_{2k-1} = 0 \quad (k=1, 2, \ldots, n. \ k \neq m)
\]

(5)
As mentioned above, \(4n - 1\) equations are defined from equations (1), (3) and (5). Because the number of unknowns is \(4n\) \((a_1, \ldots, a_{2n}, b_1, \ldots, b_{2n})\), we can solve the system of equations by normalizing other unknowns using one certain unknown. Here, we choose \(a_{2m-1}\) as the unknown to normalize and represent the normalized unknowns using the symbol "\(^\wedge\)". In this case, the reflection coefficient at Port: \(m\), \(S_{m,m}\), and the transmission coefficient from Port: \(m\) to Port: \(k\), \(S_{k,m}\), are provided by the following equations.

\[
S_{m,m} = \frac{-1 - \hat{b}_{2m-1} + 2\hat{b}_{2m}}{-1 + 2\hat{a}_{2m} - \hat{b}_{2m-1}} \quad (m=1, 2, \cdots, n) \quad (6)
\]

\[
S_{k,m} = \frac{-\hat{a}_{2m-1} - \hat{b}_{2m-1} + 2\hat{b}_{2m}}{-1 + 2\hat{a}_{2m} - \hat{b}_{2m-1}} \quad (m=1, 2, \cdots, n. \quad k=1, 2, \cdots, n. \quad m \neq k) \quad (7)
\]

### 2.2 Radiation pattern

If the antenna except ANT \(m\) (under the test) is terminated, we obtain the following equations from equation (1).

\[
b_{2m-1} = S_{2m-1,2m-1}a_{2m-1} + S_{2m-1,2m}a_{2m} \quad (m=1, 2, \cdots, n) \quad (8)
\]

\[
b_{2m} = S_{2m,2m-1}a_{2m-1} + S_{2m,2m}a_{2m} \quad (m=1, 2, \cdots, n) \quad (9)
\]

Eliminating \(b_{2m-1}\) and \(b_{2m}\) from equations (8) by using equation (3), the following equation can be obtained.

\[
\alpha = \frac{a_{2m}}{a_{2m-1}} = \frac{1 - S_{2m-1,2m-1}}{1 - S_{2m,2m-1}} \quad (m=1, 2, \cdots, n) \quad (10)
\]

If radiation patterns are measured under the above conditions, they can be obtained with a cancelled current on the outside of the coaxial cables. Equation (9) means that two ports, i.e., Port: 2\(m\)-1 and Port: 2\(m\), are excited by \(a_{2m-1}\) and \(a_{2m}\). During actual measurements, it is difficult to excite both ports with an arbitrary amplitude and phase. The offline synthesis of array-element patterns for each port is a useful and practical method. \(E_{2m-1}\) and \(E_{2m}\) are defined as array-element patterns for Port: 2\(m\)-1 and Port: 2\(m\), respectively, and a synthesized array pattern \(E_i\) can be obtained as follows.

\[
E_i = E_{2m-1} + \alpha E_{2m} \quad (m=1, 2, \cdots, n) \quad (11)
\]

\(E_i\) is the radiation pattern with a cancelled current on the outside of the coaxial cables.

### 2.3 Actual gain

When two ports are excited by \(a_{2m-1}\) and \(a_{2m}\), the reflected waves \(b_{2m-1}\) and \(b_{2m}\) can be described as follows.

\[
b_{2m-1} = (S_{2m-1,2m-1} + S_{2m-1,2m})a_{2m-1} = \beta a_{2m-1}
\]

\[
b_{2m} = (S_{2m,2m-1} + S_{2m,2m})a_{2m} = \gamma a_{2m}
\]

To subtract the reflected power from the incident power for both ports results in the total incident power, \(P_{in}\).

\[
P_{in} = |a_{2m-1}|^2 + |a_{2m}|^2 - |\beta a_{2m-1}|^2 - |\gamma a_{2m}|^2 = |a_{2m-1}|^2 \left| 1 + |\beta|^2 - |\gamma|^2 \right|
\]

(12)

If a standard gain antenna with a gain of \(G_a\) is connected to Port: 2\(m\)-1, the input power to the antenna, \(P_{ind}\) becomes \(P_{ind} = |a_{2m-1}|^2\). If \(E_d\) is the observed level for the standard gain antenna, the absolute gain of the antenna under the test, \(G_{act}\), can be expressed as follows.

\[
G_{act} = \frac{P_{ind}}{P_{in}} \quad G_{ind} = G_a \quad G_{act} = \frac{E_d}{P_{ind}} \quad \frac{|E_d|}{|P_{ind}|} = \frac{E_d}{P_{ind}} \quad \frac{|E_d|}{|P_{ind}|} = \frac{E_d}{P_{ind}}
\]

(13)

Because the gain calculated by equation (13) is normalized by the actual power supplied to the antenna during the test, mismatch loss is not taken into account. \(G_a\), divided by the mismatch loss \(M\) (positive in dB expression) results in an actual gain \(G_{act}\), which can be expressed as follows,

\[
G_{act} = \frac{G_a}{M} = \frac{G_a}{M} \left( 1 - \frac{Z_m - Z_0}{Z_m + Z_0} \right)
\]

(14)

where \(Z_m\) is the input impedance of ANT \(m\) and \(Z_0\) is the characteristic impedance.
3. Measurement results

3.1 Impedance and mutual coupling

Measured impedance characteristics ($S_{11}$) and mutual coupling ($S_{21}$) for the model of Fig. 3 are shown in Fig. 4. The measured frequency range is from 700MHz to 900MHz. For comparison, the measurement results obtained using the proposed method and those obtained using the conventional method (connect coaxial cable directly) are shown in the same figure. Concerning the reflection characteristics, it turns out that resonance frequency is drastically shifted from near 800 MHz to the low frequency using the conventional method, and a correct measurement has not been performed. Meanwhile, the results using the proposed method are in good agreement with the calculated results, and the measurement accuracy is improved.

3.2 Radiation characteristics

The measured radiation pattern for the model of Fig. 5 is shown in Fig. 6. The patterns are normalized by the peak value in the z-x plane. The result obtained using the proposed method provides close agreement with the calculated result. On the other hand, the results of the conventional method are influenced by the cable connected to the antenna, and the degree of coincidence is lower than that of the proposed method.

Figure 7 shows the frequency characteristics of the average of actual gain in the z-x plane. The frequency is normalized by the resonant frequency. For reference, the result obtained using the measurement system of fiber optics [1] is shown in Fig. 7. An o/e converter required for measurement by fiber optics is put in a small metal case on the ground plane. Because the resonant frequency is shifted due to the influence of the metal case, the input impedance of the antenna is tuned by adjustment of the antenna length and matching circuit. Both results shown in Fig. 7 are in good agreement and the validity of the results obtained using the proposed method is confirmed.

4. Conclusion

We have proposed an S-parameter method including radiation characteristics and mutual coupling of antennas on a small ground plane. We have also shown that accurate measurement results can be obtained using the proposed method.

References

Figure 3: The measurement model for S-parameter between two antennas.

Figure 4: Measurement results of S-parameter between two antennas.

Figure 5: The measurement model for radiation pattern.

Figure 6: Measurement results of radiation pattern.

Figure 7: Measurement results for antenna gain.