1. Introduction

The Friis transmission formula between two antennas is used to measure the gains of these antennas. Generally the gains obtained by the Friis formula at a finite distance are not the far-field gains because the far-field gain is defined to be a value at infinite distance. The gain at the finite distance is called as the near-field gain. To obtain the far-field gain from the near-field gain in measurement is an important topic [1].

Recently the concept of the phase centers of antennas is effectively used in the far-field gain measurement at finite distance [2]. Because the phase center is closely related to the amplitude center of an antenna [3], the far-field gains are obtained by the Friis formula using the distance between the amplitude centers of the antennas in good approximation. However, we have not known papers why the concept of the amplitude centers applied to the Friis formula gives the far-field gain.

We will show the reason clearly by using the expression derived from the Kern transmission formula [4]. As an example, we will show a case of standard horn antenna at R band.

2. Phase Centers, Amplitude Center, and Kern Transmission Formula

The phase centers of a few kinds of horn antennas are defined and calculated in [5]. The phase centers are defined using the far-field pattern around the boresight and have different values in the E and H planes.

Figure 1 shows the phase difference between the phase center and the aperture at the wavenumber $k$ direction. The phase delay relative to the origin ($z = 0$) is given as

$$\varphi = -kd \cos \theta.$$  

(1)

Figure 1: Relation between phase center and phase difference.

Using the Kern transmission formula [4], the $S$-parameter $S_{21}$ from the antenna 1 to the antenna 2 (assuming a reciprocal antenna) is expressed as

$$S_{21}(z) = \frac{\gamma_0}{\eta_0 k} \int \gamma_s s_{20}(-K) \cdot s_{10}(K) e^{-j\gamma z} dK$$  

(2)
where $\mathbf{K}$ and $\gamma$ are the $xy$ and $z$ components of $\mathbf{k}$ respectively, $s_{20}(\mathbf{K})$ and $s_{10}(\mathbf{K})$ are the complete transmission characteristics of the antenna 1 and 2. Converting the integral variable $\mathbf{K}$ to $(\tau, \phi)$ as (17) in [1], (2) reduces to

$$S_{21}(z) = \frac{\gamma_0}{\eta_0} \int_0^1 U \left( \sqrt{\tau(1-\tau)} \right) e^{jkz \tau} d\tau$$

(3)

where the evanescent components of the complete transmission characteristics are omitted and

$$U(K_n) \equiv \int_0^{2\pi} y^2 s_{20}(-\mathbf{K}) \cdot s_{10}^{\circ} d\phi$$

$$= \int_0^{2\pi} \{ E_\theta(\pi-\theta, \phi)E_\theta(\theta, \phi) - E_\phi(\pi-\theta, \phi)E_\phi(\theta, \phi) \} d\phi$$

(4)

and

$$\tau \equiv \frac{k-\gamma}{k} = 1 - \sqrt{1-K_n^2} = 1 - \cos \theta.$$  

(5)

In (4), we assume that the antenna 1 and 2 are identical, $E_\theta(\theta, \phi)$ and $E_\phi(\theta, \phi)$ are $(\theta, \phi)$ components of the far-field patterns of the antennas.

As shown in [1], if $kz >> 1$, the main contribution of the integrand in (3) comes around $\tau = 0$ ($K_n = 0$) or $\theta = 0$. Therefore, extracting the phase center terms including $dE$ and $dH$ of the $E$ and $H$ planes and integrating the phase term of the integrand of (4) from 0 to $2\pi$, we obtain

$$U(\tau) = U(0)e^{j2kd(1-\cos \theta)} U_n(K_n)$$

(6)

where $U_n(0)$ is almost a real quantity around $K_n = 0$ and $d=0.5(d_{E}+d_{H})$. Inserting (6) into (3), we obtain

$$S_{21}(z) = \frac{\gamma_0}{\eta_0} \left( U(0) \int_0^1 U_n \left( \sqrt{\tau(1-\tau)} \right) e^{jk(z+2d)\tau} d\tau \right)$$

(7)

and expand in $z+2d$ using integration by parts as

$$S_{21}(z) = \frac{\gamma_0}{\eta_0} \left[ \frac{-1}{2j(z+2d)} \left[ 1 - \frac{U_n'(0)}{jk(z+2d)} + O \left( \frac{1}{(z+2d)^2} \right) \right] \right]$$

(8)

where we assume $U_n(0)=1$ and $U_n(1)=U_n'(1)=0$.

For the identical antennas, the Friis formula at the aperture distance $z$ can be written as

$$|S_{21}(z)| = M \frac{\lambda}{4\pi z} G(z)$$

(9)

where $G(z)$ is the near-field gain at $z$ and $M$ is the mismatch factor. On the other hand, noting that $U_n'(0)$ is real, (8) can be expressed as

$$|S_{21}(z)| = M \frac{\lambda}{4\pi (z+2d)} G(\infty) \left[ 1 + O \left( \frac{1}{(z+2d)^2} \right) \right]$$

(10)

Comparing (9) and (10), we find that we can obtain the far-field gains using the Friis transmission formula with the distance between the amplitude centers of the antennas instead of the distance between the apertures. The order of the error is $O((z+2d)^2)$ which is equal to that when using the amplitude centers in [5]. That is, we have clearly proved that the average of the phase center
locations in the E and H plane is the same as the amplitude center location in the Friis transmission formula.

3. Phase Centers of R-Band Standard Horn Antenna

To check (6), we calculate the far-field pattern of a R-band standard horn antenna shown in Fig. 2. The horn antenna models the MI-12-1.7 manufactured by MI Technologies. The aperture sizes are 367.5 mm (H plane) x 272 mm (E plane) and the height from the aperture to the junction of the waveguide (WR430) is 271 mm. The waveguide sizes are 109.2 mm x 54.6 mm in cross-section and 150 mm in length. The excitation mode is TE10 and polarized along the x axis.

Figure 2: R-band standard horn antenna and its normalized pattern in dB scale at 2.45 GHz.

The far-field pattern is calculated by FEKO (an electromagnetic simulator by EM software & systems) at 2.45 GHz. The phase variations of the patterns around $\theta = 0$ are shown in Fig. 3. The phase variation (Phase.H) in the H plane is larger than that (Phase.E) in the E plane. Using (1), the phase variations (Phase.H.dH and Phase.E.dE) for $d_H = 0.133$ m and $d_E = 0.090$ m are also shown in Fig. 3. The average ($0.5*(\text{Phase.H}+\text{Phase.E})$) of the phase variations in the E and H planes is shown in dots. Using (4), the phase variation ($0.5*\text{Phase.Un}$) and the amplitude variation ($\text{Amp.Un.dB}$) are calculated. The phase variation ($\text{Phase.dA}(0.113 \text{ m})$) for $d = 0.113$ in (6) is shown to be exactly the average of those for $d_H$ and $d_E$. Since the phase variation of Un includes that of the two identical horn antennas as indicated in (4), the half of the phase variation is compared in Fig. 3.

Figure 3: Phase variations and amplitude variation of Un.
The real and imaginary parts of $U_n$ in (6) are shown in Fig. 4. From $\theta = 0$ to 10 degrees, the imaginary part is almost zero. This means that the phase variation is well approximated by $2k\delta (1-\cos \theta)$. In addition, $U_n$ is almost zero over $\theta = 50$ and $U_n(1)= U'_n(1)=0$ are satisfied assumed to derive (8).

![Figure 4: Real and imaginary parts of $U_n$.](image)

4. Conclusion

We have derived the amplitude center location in the Friis transmission formula from the Kern transmission formula using the phase centers of the antennas. The final expression (10) clearly shows that the Friis transmission formula with the distance between the amplitude centers of the antennas can be used to measure the far-field gains of the antennas in good approximation.

The assumptions used in the derivation are validated by the simulation of a R-band standard horn antenna.

We are now investigating (7) further and will show the limitation and the extension of the amplitude center for antenna measurements in another paper.

References