Application of Optimized Sparse Antenna Array in Near Range 3D Microwave Imaging

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1. Introduction

Near range three-dimensional (3D) microwave imaging techniques have broad application prospects in the field of concealed weapon detection, biomedical imaging, nondestructive testing, etc[1]. In this paper, the technique of optimized sparse antenna array is applied to near range 3D microwave imaging, which can greatly reduce the complexity of imaging systems.

In detail, the 3D geometry in near range microwave imaging can be formed by scanning a planar aperture in two orthogonal directions and transmitting broadband signals in range direction. The planar aperture is generally synthesized by a line antenna array scanning along the orthogonal direction. However, the antenna array should contain a large number of elements to ensure the aperture length. To reduce the number of real array elements and lower the system complexity, the technique of optimized sparse antenna array can be used. In detail, by appropriately deploying sparse array and designing combinations of radiating antennas and receiving antennas, a sparse antenna array can be equivalent to a filled array. By analyzing the relationship between transceiver combinations and the convolution principle, the author develops a method of arranging optimized sparse array in [2].

This paper focuses on the application of optimized sparse array in 3D imaging geometry. When sparse array is adopted, the phase error caused by range difference between equivalent channel and real channel should be considered. Therefore, this paper establishes the echo model with the error and deduces 3D back-projection (BP) algorithm with the compensation of the error. The structure of the paper is as follows. 3D sparse array imaging geometry and echo model are established in Section 2, in which the geometry is formed by transmitting broadband signals in range direction and arranging sparse antenna array which is set along elevation and scans along azimuth; In Section3, the paper deduces the BP algorithm applied to 3D sparse array geometry, where the optimized sparse antenna array is solved through the method in [2]; After that, the imaging geometry and imaging algorithm are investigated and verified by means of near range experiment in Section 4; Finally, Section 5 draws several conclusions.

2. Near Range 3D Sparse Array Imaging Geometry

By introducing the technique of optimized array to near range imaging, this section establishes the 3D sparse array geometry and corresponding echo model.

2.1 Optimized sparse antenna array

Using the optimized method of [2], we can derive the optimized sparse antenna array described as the symmetric vector in Fig.1. And the The real element number of the sparse array is $M = 2q + m$, $m = \lceil \frac{L-2q}{q} \rceil$, where $\lceil \cdot \rceil$ means rounding up.

![Figure 1: Location vector of optimized sparse array.](image)
2.2 3D sparse array geometry and corresponding echo model

Based on the sparse array in 2.1, 3D sparse array imaging geometry can be established as Fig.2, the azimuth and range direction are defined as axis X and Y, and the elevation direction along which the sparse antenna array is arranged is denoted as axis Z.

\( \textbf{A}_t \) and \( \textbf{A}_r \) indicate the locations of radiating and receiving antennas. This transceiver couple can be equivalent to an antenna \( \textbf{A}_e \), and the location of this equivalent antenna is called equivalent phase center. \( \textbf{I} \) indicates a point scatterer at \( (x, y, z) \). The distance between radiating antenna and \( \textbf{I} \) is \( R_t \), the distance between receiving antenna and \( \textbf{I} \) is \( R_r \), \( y_r \) is the distance of scene center from the planar aperture.

![Figure 2: 3D sparse array imaging geometry.](image)

Suppose that the 3D complex reflectivity function of the distributed targets in Cartesian coordinates is \( I(x, y, z) \). In this case, the echo from a given point scatterer located at \( (x, y, z) \), which is measured via transceiver combination \( \textbf{A}_t \textbf{A}_r \), can be written as

\[
E_I(\textbf{A}_t, \textbf{A}_r, K_\omega) = I(x, y, z) \exp[-jK_\omega (R_t + R_r)]
\]

where \( K_\omega \) is frequency wave-number.

When the transceiver channel of \( \textbf{A}_t \) and \( \textbf{A}_r \) is equivalent to \( \textbf{A}_e \), the range history of real channel is \( (R_t + R_r) \), while the range history of equivalent channel is \( 2R_e \). The phase error caused by the range difference is,

\[
\Delta \varphi = (R_t + R_r) - 2R_e
\]

The error is related to transceiver channel and target positions, so it should be compensated referring to each transceiver channel and each target point.

In summary, the response measured by the transceiver combination \( \textbf{A}_t \textbf{A}_r \) can be rewritten as,

\[
E(\textbf{A}_t, \textbf{A}_r, K_\omega) = \int \int \int_V I(x, y, z) \frac{\exp[-jK_\omega (R_t + R_r)]}{R_t R_r} d\mathbf{r} = \int \int \int_V I(x, y, z) \frac{\exp[-jK_\omega (2R_e + \Delta \varphi)]}{R_t R_r} d\mathbf{r}
\]

where \( V \) denotes the illuminated area, \( \mathbf{r} \) is position vector of the targets and \( d\mathbf{r} = dx dy dz \).

3. BP Algorithm Applied to 3D Sparse Array Geometry

This section deduces the BP algorithm applied to 3D sparse array geometry, which includes the procedures of inconsistency compensation, equivalent phase error compensation and range decay compensation. The process of BP algorithm, which is applied to near range 3D sparse array imaging geometry, includes several steps as follows. The block diagram is summarized in Fig.3.

1). Compensate the inconsistency error of antenna elements \( m_t \varphi_t, m_r \varphi_r \). For the amplitude and phase error caused by inconsistency of antenna elements will degrade the imaging result, the first step is the compensation of this error. Denote the echo after compensation as \( E'(\textbf{A}_t, \textbf{A}_r, K_\omega) \).

2). Calculate the coordinates and the number of pixels \( N \) of the 3D complex reflectivity image, which corresponds to the illuminated area \( V \).
3). Based on 2), \(E' (A_t, A_r, K_\omega)\) can be expressed as,

\[
E' (A_t, A_r, K_\omega) = \int \int \int_V I(x, y, z) \frac{\exp \left[ -jK_\omega (2Re + \Delta \varphi) \right]}{R_t R_r} d^3r = \sum_{n}^{N} I(x_n, y_n, z_n) \frac{\exp \left[ -jK_\omega (2Re + \Delta \varphi) \right]}{R_t R_r}
\]  

(4)

4). Compute the filter function \(H_0 (A_t, A_r, n)\) which corresponds to echo \(E' (A_t, A_r, K_\omega)\) and the \(n_{th}\) pixel \(I(x_n, y_n, z_n)\) of the 3D complex image. To compensate phase error between real channels and their equivalent channels and the range decay, the filter function can be written as,

\[
H_0 (A_t, A_r, n) = R_t R_r \cdot \exp \left[ jK_\omega (2Re + \Delta \varphi) \right]
\]  

(5)

5). Multiply \(E' (A_t, A_r, K_\omega)\) with \(H_0 (A_t, A_r, n)\), calculate the integral with respect to \(K_\omega\) and sum the integral result with respect to elevation and azimuth. Assuming the number of equivalent phase centers is \(P\) which represents the elevation sampling points, and assuming the azimuth sampling points is \(Q\), the imaging result \(I(x_n, y_n, z_n)\) of the \(n_{th}\) pixel can be written as,

\[
I(x_n, y_n, z_n) = \sum_{p}^{P} \sum_{q}^{Q} \int_{K_\omega}^{K_\omega} E' (A_t, A_r, K_\omega) H_0 (A_t, A_r, n) dK_\omega
\]  

(6)

6). Compute the imaging result of each pixel in illuminated area through 5). We can obtain the 3D complex reflectivity image result of the whole illuminated area.

**Figure 3:** Block diagram of BP algorithm for 3D sparse array geometry.

### 4. Near Range Experiment

This section verifies the effectiveness of the geometry and imaging algorithm of this paper.

#### 4.1 Experiment parameters and solved sparse array

Assume the system send step frequency signal range from 30 to 36 GHz with 801 points, then, the center wavelength is 0.0091 m. In azimuth, the physical size of element is 0.012 m, scanning points is 165 and scanning interval is 0.005 m. In elevation, physical size of element is 0.005 m, the number of filled array elements \(L\) is 55 and the interval of equivalent phase centers is 0.0085 m.

Based on the method of [2] and the parameters, the solved sparse array includes 19 real elements which can generate 109 equivalent phase centers which is shown in Fig.4.

**Figure 4:** The top one is the filled array and bottom one shows solved sparse array.

Fig.5(a) shows the experimental geometry. There is an antenna array along elevation. The main targets include 8 solid metal spheres of 2cm diameter and a trihedral corner reflector with 20cm short sides. The 8 spheres form a cuboid of 0.2 m in range, 0.15 m in azimuth and 0.2 m in elevation, which is 0.7 m away from the planar aperture. The trihedral is 4.95 m away from the planar aperture.
4.2 Imaging result

In order to analyze the equivalent error, both of the results reconstructed via algorithms without and with equivalent phase error compensation are obtained, which are shown in Fig. 5.

Fig.5(b) and (c) are the results of the imaging algorithm without equivalent phase error compensation which ignores the phase error between real channels and their equivalent channels, resulting in that the targets cannot be focused in elevation direction; however, the algorithm of this paper computes different filter functions which are related to different transceiver combinations and different target locations, therefore, the trihedral and metal spheres can be well focused in Fig.5(d) and (e).

![Figure 5: Experimental geometry and imaging result.](image)

The experiment and the imaging results of this subsection further validate the feasibility and correctness of the proposed 3D sparse array imaging geometry and corresponding BP algorithm.

5. Conclusions

By applying the techniques of optimized sparse array to near range imaging, the paper establishes near range 3D optimized sparse array imaging geometry. Then, BP algorithm applied to the geometry is deduced. Finally, the paper verifies the feasibility and correctness of the imaging geometry and corresponding algorithm by means of near range imaging experiments. The imaging results of experiment demonstrate that 3D complex image of illuminated scene can be well reconstructed via the 3D optimized sparse array geometry and corresponding algorithm of this paper.

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