Fully Isotropic Singularity-Spreading Phase Unwrapping

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1. Introduction

Digital Elevation Model (DEM) has various applications in topography mapping and land deformation detection. Phase interferogram represents difference of distances from spatially or temporally different radars to a target. To generate DEM, the key technique is phase unwrapping since phase differences are wrapped as the principal values between $-\pi$ and $\pi$ [1,2].

However, interferograms are distorted by various harmful factors such as interference and attenuation during propagation, diffraction and scattering/reflection at ground surface. It is widely known that there are many singular points (SPs), which hinders determining the absolute phase values. Concerning SPs, we reported that the amplitude of SPs and their vicinities are almost zero, and phase values are distorted [3,4]. In addition, most of SPs are found to be pairs of closely located positive and negative singularity.

According to these facts, we have proposed singularity-spreading phase unwrapping (SSPU) method based on the idea that the SPs should be compensated in a continuous manner with their vicinities. This method has ability to remove positive/negative singularity by making use of phase compensator with a calculation cost almost independent of the density of SPs [5]. Contrarily, traditional approaches estimate phase-unwrapping path with a calculation cost proportional to the number of SPs [6,7]. However, recent studies confirmed that the spreading manner of the conventional SSPU has a problem that the spreading occurs in a checkered pattern, resulting in a slow convergence and unnecessarily large extension.

In this paper, we propose fully isotropic singularity-spreading phase unwrapping (FI-SSPU) which combines conventional spreading with diagonal spreading so as to solve this problem.

2. The Fully Isotropic Singularity-Spreading Phase Unwrapping

2.1 Four-directional Singularity-Spreading Phase Unwrapping (SSPU)

First, we present the processing algorithm of conventional four-directional SSPU. According to the concept previously mentioned, rotation $R(i, j)$ is defined obtained by positionally discrete phase data $\phi(i, j) [(i, j) \in S_0, S_0 := \{(i, j) | i = 1, 2, \cdots, M \land j = 1, 2, \cdots, N\}]$ in an interferogram.

\[
R(i, j) = \frac{1}{2\pi} \{ -\Delta \phi_1(i, j + 1) + \Delta \phi_2(i, j) + \Delta \phi_3(i, j) - \Delta \phi_4(i + 1, j) \} \tag{1}
\]

\[
\Delta \phi_a(i, j) = pv[\phi(i + 1, j) - \phi(i, j)], \quad (i, j) \in S_1
\]

\[
\Delta \phi_{b}(i, j) = pv[\phi(i, j + 1) - \phi(i, j)], \quad (i, j) \in S_2
\]

where $pv[\cdot]$ stands for principal value and $S_1 := \{(i, j) | i = 1, 2, \cdots, M - 1 \land j = 1, 2, \cdots, N\}, S_2 := \{(i, j) | i = 1, 2, \cdots, M \land j = 1, 2, \cdots, N - 1\}, S_3 := \{(i, j) | i = 1, 2, \cdots, M - 1 \land j = 1, 2, \cdots, N - 1\}$ for an $M \times N$ interferogram.

We previously proposed a phase unwrapping method shown in Fig.1. First, we spread the phase singularity at respective SPs to their vicinities in the four directions in the phase difference domain (1) by adding the compensator values (2) calculated below obtained as the rotation values (2). This process generates the 1st compensated interferogram. We repeat the above spreading process for $k$ times to obtain a $k$th compensated interferogram until the maximum singularity in the $k$th interferogram becomes almost zero so that we can unwrap the interferogram just by integrating the phase difference with any integral lines. However, the spreading in the conventional four-directional SSPU occurs in a back-and-forth manner.
∆ϕ(k) (i, j) + 1) = −2π R(k)(i, j) / 4, (i, j) ∈ S

Δϕcy (i, j) = +2π R(k)(i, j) / 4, (i, j) ∈ S1

Δϕcy (i + 1, j) = +2π R(k)(i, j) / 4, (i, j) ∈ S2

ΔΦcy (i, j) = \sum_{k=1}^{l} Δϕcy (i, j), (i, j) ∈ S1

ΔΦcy (i, j) = \sum_{k=1}^{l} Δϕcy (i, j), (i, j) ∈ S2

Figure 1: Flowchart of SSPU (conventional and proposed).

as shown below in the experiment section. Then the process yields a checkered pattern in the residual rotation. Consequently, the spreading shows a slow convergence and unnecessarily large extension.

Though this conventional spreading process spreads the singularity only to four directions, the residual rotation is spread to the whole directions in multiple iterations. Nevertheless, this method has a weakness that the positive and negative SPs cannot cancel out directly under a certain condition.

That is, when we take discrete pixel sets S4 ∈ S3, S4 := [(i, j) | (odd, odd) \lor (even, even)] and S5 ∈ S4, S5 := [(i, j) | (even, odd) \lor (odd, even)], rotations in S4 at k th iteration R(k)(i, j), (i, j) ∈ S4 are spread to S5 (R(k+1)(i, j), (i, j) ∈ S5). Conversely, rotations in S5 at k th iteration R(k)(i, j), (i, j) ∈ S5 are spread to S4 (R(k+1)(i, j), (i, j) ∈ S4). Then a SP in S4 does not cancel out with an opposite SP in S5 even if they are very close to each other.

2.2 Proposal: Fully Isotropic SSPU

To solve the above problem, we propose fully isotropic singularity-spreading phase unwrapping (FI-SSPU). This method has a rapid convergence. Thus, smaller spreading extension and lower low-frequency distortion are accomplished. Fig.2 is the schematic of the process. In this method, compensators at kth iteration obtained by a residual rotation are defined as

Δϕcx (i, j) = +2π R(k)(i, j) / 4, (i, j) ∈ S1

Δϕcx (i \pm 1, j) = +2π R(k)(i, j) / 24, (i, j) ∈ S1

Δϕcx (i, j + 1) = −2π R(k)(i, j) / 4, (i, j) ∈ S1

Δϕcx (i \pm 1, j + 1) = −2π R(k)(i, j) / 24, (i, j) ∈ S1

Δϕcy (i, j) = −2π R(k)(i, j) / 4, (i, j) ∈ S2

Δϕcy (i, j \pm 1) = −2π R(k)(i, j) / 24, (i, j) ∈ S2

Δϕcy (i + 1, j) = +2π R(k)(i, j) / 4, (i, j) ∈ S2

Δϕcy (i + 1, j \pm 1) = +2π R(k)(i, j) / 24, (i, j) ∈ S2.

Although conventional process spreads the residual rotation only to four directions, proposed process spreads the rotation to totally eight directions at each iteration, thereby solving orthogonal spreading problem. After sufficient number of iterations, the rotational component of phase becomes almost zero. Therefore, the two-dimensional unwrapped phase can be easily and uniquely determined, independent of the integration path.
3. Experiments and Results

We present experimental results of the conventional four-directional SSPU and the proposed FI-SSPU to compare these methods in terms of spreading process and convergence. We apply these methods to interferograms made by regular 16-look mean-filtered process for the following two areas. The InSAR data were generated from the observation by JERS-1 satellite.

Area1 (including Area2) : 256×256 pixel area : Lake yamanaka and mountain ridges, SPs : 528 positives and 529 negatives (Fig.3(a)(b)).

Area2 : 64×64 pixel area : south of Lake yamanaka and mountain ridges, SPs : 14 positives and 14 negatives (Fig.3(c)(d)).

Fig.4 briefly shows the advantage of proposed algorithm. In the result of proposed method (Fig.4(b)), spreading efficiency is higher than the conventional (Fig.4a). Therefore number of iteration and spreading extension is smaller when the maximum residual rotation is reduced to $R^{(k)} \approx \pi/10$ equally for both of them.

Fig.5 shows (a) x- and (b) y-directional compensators and (c) residual rotation when the spreading process is finished. With these compensators, residual rotations become almost zero, as shown in Fig.5(c).
The double logarithmic plot in Fig.6 illustrates the convergence in conventional and proposed spreading processes as maximum residual rotations versus iterations. In the proposed FI-SSPU, spreading process converges in 1/5 to 1/50 times iterations.

4. Conclusion

We have proposed the FI-SSPU method that spreads SPs omnidirectionally by means of eight-directional compensators with a calculation cost almost independent of the density of SPs.

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References