Anisotropic Photonic Band Gaps in Three-dimensional Magnetized Plasma Photonic crystals

Hai-Feng Zhang\(^1,2\), Shao-Bin Liu\(^1\), Huan Yang\(^1\)

\(^1\) College of Electronic and Information Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing China
\(^2\) Nanjing Artillery Academy, Nanjing 211132, China
Email: hanlor@163.com

Abstract- The band Faraday effects in photonic band gaps (PBG) for the three-dimensional (3D) magnetized plasma photonic crystals (PCs) consisting of the uniaxial material with face-centered-cubic (fcc) lattices are theoretically investigated by a modified plane wave expansion (PWE) method, which the homogeneous anisotropic dielectric spheres (the uniaxial material) immersed in the magnetized plasma background, as the Faraday effects of magnetized plasma are considered. The anisotropic PBGs and a flatbands region can be obtained. The effects of the anisotropic dielectric filling factor on the properties of first two anisotropic PBGs are investigated in detail. The numerical results show that the anisotropy can open partial band gaps in fcc lattices at \(U\) and \(W\) points, and the complete PBGs can be achieved compared to the conventional 3D dispersive PCs composed of the magnetized plasma and isotropic material. It also is shown that the first two anisotropic PBGs can be tuned by the filling factor.

I. INTRODUCTION

Photonic crystals (PCs) have been attracting a lot of interest since the concept was firstly proposed by Yablonovitch [1] and John [2]. The PCs is a kind of artificial material with a periodically modulated dielectric constant in space, and can produce the magic regions named photonic band gaps (PBGs) [3], where the propagation of electromagnetic wave (EM wave) is forbidden. Thus, PCs can be used to design the applications in the optics. Recently, various dispersive materials have been introduced into PCs to obtain the tunable PBGs, such as ferrofluids [4], plasma [5], superconductor [6], and metal [7]. The tunable PBGs means the PBGs can be manipulated by the different stimuli, such as the magnetic field, temperature or electric field, and not need change the topology of PCs. Investigating of the tunable PCs becomes a new research focus. The plasma photonic crystals (PPCs) is a typical of tunable PCs, which is recently a hot research area. The idea of PPCs was firstly proposed by Hojo and Mase [8], and can be looked as metamaterial [9]. The PPCs also can be used to design some novel tunable devices [10-12] which can realize in the microwave region. Therefore, the PPCs have been extremely investigated in detail. Compared to the conventional PCs, the PPCs can display strong spatial dispersion [13, 14] and also can be tuned by the external magnetic field [15]. If the external magnetic field is introduced into the PPCs, the magnetized plasma photonic crystals (MPPCs) will be realized. As we know, if the external magnetic field is introduced into the plasma, two kinds of magneto-optical effects can be achieved. One configuration, in which the external magnetic field is perpendicular to the EM wave vector, gives rise to so-called Voigt effects. The other is that the EM wave vector is parallel to the external magnetic field. In this case, the Faraday effects can be obtained [16]. If different magneto-optical effects of magnetized plasma are considered, the different dispersive properties of MPPCs will be obtained. Recently, the properties of MPPCs have intensively been studied in theory and experiment by many research groups [17-20].

The most works on MPPCs are 1D or 2D cases. A few published reports about 3D MPPCs until Zhang et al. [21, 22] investigated the dispersive properties of 3D MPPCs with simple-cubic and diamond lattices as the Faraday effects of magnetized plasma are considered. As mentioned in their works, these 3D MPPCs suffer from high symmetry and dielectric constant of dielectric must be sufficiently large so that the resonant scattering of EM waves is prominent enough to open a band gap [23]. Unfortunately, technological difficulties in fabricating the 3D MPPCs with the large dielectric constant of dielectric can be found to achieve the complete PBGs. Therefore, if we want to obtain the complete PBGs in 3D MPPCs, the symmetry reduction [24] and anisotropy in dielectric may be good choices [25]. We note that many previous studies on the MPPCs considered that the filling dielectric only is isotropic, and the Faraday effects of magnetized plasma are not considered for 3D case at same time. Therefore, the aim of the present paper is to perform a study of the band Faraday effects in the 3D dispersive PCs consisting of the magnetized plasma and uniaxial material with fcc lattices based on a modified PWE method. The proposed 3D MPPCs are that the anisotropic dielectric spheres are immersed in the magnetized plasma background periodically with fcc lattices.

II. THEORETICAL MODEL AND FORMULATION

As we know, the expression of the plasma function \(\varepsilon_p\) is determined by the angle \(\theta\) between the wave vector and the external magnetic field [16, 17]. If the Faraday effects of magnetized plasma are considered (\(\theta=0\) ), the dielectric function of magnetized plasma is anisotropic and has this form:

\[
\varepsilon_p(\omega) = 1 - \frac{\omega_p^2}{\omega^2 - j\nu_c \omega \pm \omega_0} \tag{1}
\]

where \(\omega_p\), \(\nu_c\), and \(\omega_0\) are the plasma frequency, the plasma collision frequency and plasma cyclotron frequency, respectively. Plasma frequency \(\omega_p = (\varepsilon' n_e / \varepsilon_m)^{1/2}\) in which \(\varepsilon\), \(n_e\) and \(\varepsilon_m\) are electric quantity, electric mass, plasma density and dielectric constant in vacuum, respectively.
plane waves are used to calculate, the \( \gamma \) and \( \delta \) respectively. In order to consider the ordinary materials, which can be found in the nature. The uniaxial material is one kind of anisotropic material, which can be found in the nature. The uniaxial material has two different principal-refractive indices named ordinary-refractive and extraordinary-refractive indices, and we consider the ordinary-refractive and extraordinary-refractive indices are \( n_o \) and \( n_e \) respectively. For the uniaxial material, the dielectric constant \( \varepsilon \) is a dyadic and can be written as

\[
\varepsilon = \begin{pmatrix}
\varepsilon_x & 0 & 0 \\
0 & \varepsilon_y & 0 \\
0 & 0 & \varepsilon_z
\end{pmatrix}
\]

where \( \varepsilon_x = n_o^2 \), \( \varepsilon_y = n_y^2 \), and \( \varepsilon_z = n_z^2 \).

Therefore, the dielectric dyadic has only three cases for diagonal element permutation as [23] (a) \( n_x^o = n_x^e, n_y^o = n_y^e, n_z^o = n_z^e \); (b) \( n_x^o = n_x^e, n_y^o \neq n_y^e, n_z^o = n_z^e \); (c) \( n_x^o \neq n_x^e, n_y^o = n_y^e, n_z^o = n_z^e \). We call them type-1, type-2 and type-3 uniaxial materials, respectively. In order to simplify, we just deduce the equations for calculating the dispersive curves of such 3D MPCCs containing the type-1 uniaxial material. According to the technique as mentioned in Ref.[21], the band structure of 3D MPCCs can be obtained from following equation and the definitions of parameter also can be found in Refs.[21, 22]:

\[
\frac{\mu}{\varepsilon} \mathbf{T} - \mu \mathbf{I} - \mu \mathbf{V} - \mu \mathbf{W} = 0 \tag{3}
\]

\[
\mathbf{T}(\mathbf{G}|\mathbf{G}) = \left( \frac{V}{c} + \frac{\omega_0}{c} \right) \delta_{\mathbf{G} \times \mathbf{G}},
\]

\[
\mathbf{U}(\mathbf{G}|\mathbf{G}) = \left\{ \sum_{i=x,y} \left( \frac{\omega_0^2}{c^2} \left( \mathbf{f} \left( 1 - f \right) \right) \left| \mathbf{G} + \mathbf{G} \right| \mathbf{F} \right) \right\} \delta_{\mathbf{G} \times \mathbf{G}},
\]

\[
\mathbf{V}(\mathbf{G}|\mathbf{G}) = \left\{ \sum_{i=x,y} \left( \frac{\omega_0^2}{c^2} \left( \mathbf{f} \left( 1 - f \right) \right) \left| \mathbf{G} + \mathbf{G} \right| \mathbf{F} \right) \right\} \delta_{\mathbf{G} \times \mathbf{G}},
\]

\[
\mathbf{W}(\mathbf{G}|\mathbf{G}) = \left\{ \sum_{i=x,y} \left( \frac{\omega_0^2}{c^2} \left( \mathbf{f} \left( 1 - f \right) \right) \left| \mathbf{G} + \mathbf{G} \right| \mathbf{F} \right) \right\} \delta_{\mathbf{G} \times \mathbf{G}},
\]

where

\[
\mathbf{M}_i = \left| \mathbf{G} \right| \mathbf{F} \times f \left( \frac{\sin \left( \left| \mathbf{G} \right| \mathbf{R} \right) + \cos \left( \left| \mathbf{G} \right| \mathbf{R} \right) \left| \mathbf{G} \right| \mathbf{R} \right), i = x, y, \]


This polynomial form can be transformed into a linear problem in 4N dimension by \( \mathbf{Q} \) that fulfills

\[
\mathbf{Q} = \mu z,
\]

\[
\mathbf{Q} = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
\]

The complete solution of Eq.(3) is obtained by solving for the eigenvalues of Eq.(4). Of course the dispersion relation can be determined by the real part of such eigenvalues. The analogue equation to Eq.(3) for another two types of cases also can be easily derived.

III. NUMERICAL RESULTS AND ANALYSIS

A symmetric set of primitive vectors for the fcc lattice is \( \mathbf{a}_1 = (0.5a, 0.5a, 0), \mathbf{a}_2 = (0, 0.5a, 0.5a), \mathbf{a}_3 = (0.5a, 0, 0.5a) \). The reciprocal lattice vector basis can be defined as \( \mathbf{b}_1 = (2\pi/a, 2\pi/a, 2\pi/a), \mathbf{b}_2 = (-2\pi/a, 2\pi/a, 2\pi/a), \mathbf{b}_3 = (2\pi/a, -2\pi/a, 2\pi/a) \). The high symmetry points have the coordinate as \( \mathbf{r} = (0, 0, 0), \mathbf{X} = (2\pi/a, 0, 0), \mathbf{W} = (2\pi/a, \pi/a, 0), \mathbf{K} = (1.5\pi/a, 1.5\pi/a, 0), \mathbf{L} = (\pi/a, \pi/a, 0), \) and \( \mathbf{U} = (2\pi/a, 0.5\pi/a, 0.5\pi/a) \). As a total number of 729 plane waves are used to calculate, the convergence accuracy is better than 1% for the lower energy bands [23]. Without loss of generality, we plot \( \omega a/2\pi c \) with the normalization convention \( \omega_p a/2\pi c = 1 \). Thus, we can define the plasma frequency as \( \omega_p = \omega_p = 0.3c/a = 0.15\omega_{pl} \) to make the problem scale-invariant. We also choose the plasma collision frequency and plasma cyclotron frequency as \( \nu_c = 0.02\omega_{pl} \) and \( \omega_e = 0.8\omega_{pl} \) respectively. Here, we only focus on the first two PBGs in the frequency domain 0–2\( \pi c/a \).

A. The anisotropic PBG for 3D MPCCs

Fig.1. Band structures for 3D fcc MPCCs with \( \nu = 0.3 \) but with different \( n_o, n_x, n_y, \) and \( \nu_x, \nu_y, \) and \( \nu_z, \) respectively. (a) \( n_x = n_y = n_z = 4.8, \omega_0 = 0, \nu_x = 0, \omega_{pl} = 0; \) (b) \( n_x = n_x = n_y = 4.8, \omega_0 = 0.15\omega_{pl}, \nu_x = 0.02\omega_{pl}, \nu_x = 0.8\omega_{pl}; \) (c) \( n_x = n_y = 4.2, n_x = n_y = 4.8, \omega_0 = 0, \nu_x = 0, \omega_{pl} = 0, \) and (d) \( n_x = n_y = 6.2, n_x = n_y = 4.8, \omega_0 = 0.15\omega_{pl}, \nu_x = 0.02\omega_{pl}, \omega_{pl} = 0.8\omega_{pl} \) respectively. The red shaded regions indicate PBGs.
In Figs.1(a) and (b), we plot the dispersive curves for 3D MPPCs with fcc lattices containing a refractive index contrast of \(n=4.8\) \((n_r=n_s=n_d=4.8)\), \(f=0.3\), but with different \(\omega_r\), \(\omega_o\), and \(\nu_c\), respectively. As shown in Fig.1(a), if \(\omega_r=0\), \(\omega_o=0\) and \(\nu_c=0\), the magnetized plasma can be looked as the air. The complete PBGs can not be found. The cases of 3D MPPCs \((\omega_r=0.15\omega_o, \nu_c=0.02\omega_o, \omega_o=0.8\omega_p)\) also can be seen in Figs.1(b). The similar conclusion also can be drawn, there are not the complete PBGs. The band structures close at \(U\) and \(W\) points in fcc lattices. Of course, the some stop band gaps can be found in the some directions of symmetry such as \(\Gamma-X\) and \(\Gamma-L\) directions. This can be explained by the relative dielectric constant of isotropic dielectric is not enough to open a band gap [23]. In order to obtain the complete PBGs, we can use the uniaxial material (anisotropic material) to replace the isotropic dielectric to form 3D MPPCs with fcc lattices. In Figs.1(c) and (d), we display the dispersive curves of 3D MPPCs doped by the type-1 uniaxial material as \(n_r=n_d=6.2\), \(n_r=n_s=n_d=4.8\) and \(f=0.3\) but with different plasma frequency, plasma cyclotron frequency and plasma collision frequency, respectively. In Fig.1(c), it is clearly seen that the two complete PBGs can be obtained as the type-1 uniaxial material is introduced. The first two PBGs cover 0.3577-0.3849 (2\(\pi/c\)) and 0.5179-0.5242 (2\(\pi/c\)), respectively. The bandwidths are 0.0272 and 0.0063 (2\(\pi/c\)), respectively. Fig.1(d) shows that the edges of PBGs are upward to higher frequencies and a flatbands region can be obtained as the Faraday effects of magnetized plasma is considered (the external magnetic field is introduced). The main reason for formed the flatbands is because the existence of surface plasmon modes which stem from the coupling effects between the plasma [21, 22]. Compare to Fig.1(a), the edges of first two PBGs shift upward to higher frequencies, and the bandwidths of first two PBGs are tuned notably. The first two PBGs span from 0.3677 to 0.3945 (2\(\pi/c\)), from 0.5264 to 0.5327 (2\(\pi/c\)), and bandwidths are 0.0268 and 0.0063 (2\(\pi/c\)), respectively. The flatbands region is located 0.12-0.2068 (2\(\pi/c\)). This can be explained by the cutoff frequency of right circular polarization and left cutoff frequency \((f_r, f_l)\) since the \(f_r\) and \(f_l\) are nearly corresponding to the lower and upper edge frequencies of flatbands region, respectively [21, 22]. As the Faraday effects are considered, the cutoff frequency of right circular polarization and left cutoff frequency \(f_r=0.2068\) (2\(\pi/c\)) \(\left(f_r=\omega_r/\sqrt{\omega_o/\omega_r}\right)\) and \(f_l=0.12\) (2\(\pi/c\)) \(\left(f_l=\omega_l/\sqrt{\omega_o/\omega_r}\right)\) [21, 22], respectively. Therefore, the flatbands are caused by the magnetized plasma itself. As mentioned above, it is found that the inclusion of type-1 uniaxial material in 3D MPPCs with fcc lattices can open band gaps near high-symmetry points and the anisotropic PBGs (complete PBGs) can be achieved compared to the 3D MPPCs containing the conventional isotropic dielectric. As the magnetized plasma is introduced in 3D dielectric PCs (the Faraday effects of magnetized plasma are considered), the PBGs can be tuned. Consequently, introducing the uniaxial material into the 3D MPPCs with high-symmetry lattices can obtained the complete PBGs.

B. Effects of the filling factor on anisotropic PBGs

In Fig.2, we plot the dependences of the properties of anisotropic PBGs for 3D MPPCs with fcc lattices containing type-1 uniaxial material on the anisotropic dielectric filling factor \(f\) as \(\omega_p=0.15\omega_o, \nu_c=0.02\omega_o, \omega_o=0.8\omega_p\), \(n_r=n_d=4.8\) and \(n_r=n_s=n_d=6.2\), respectively. As mentioned above, the filling factor on the first two PBGs for such 3D MPPCs become larger \([\nu]\) filling factor on the first two PBGs for such 3D MPPCs becomes larger, especially for \(f=0.3\). As shown in Fig.2, the first two PBGs have maximum bandwidths at \(f=0.3\) and \(f=0.4\). The both bandwidths of first two PBGs increase first then decrease with increasing \(f\). The both bandwidths of first two PBGs increase first then decrease with increasing \(f\). The both bandwidths of first two PBGs increase first then decrease with increasing \(f\). If anisotropic dielectric filling factor is less than 0.5, there does not exist the 1st PBG, and will disappear as \(f\) is larger than 0.6. The 2nd PBG will never appear until \(f\) is larger than 0.35. Thus, the first two PBGs can be tuned by the filling factor of anisotropic dielectric spheres. This can be explained in physics that increasing anisotropic dielectric filling factor means the space averaged dielectric constant of such 3D MPPCs becoming larger [21, 22]. Fig.2 also reveals that the relative bandwidth of both PBGs increase first then decrease with increasing \(f\). The maximum relative bandwidths of first two PBGs are 0.0703 and 0.0324, which can be found at the cases of \(f=0.3\) and 0.1, respectively. Compared to the case of \(f=0.35\), the frequency ranges of both PBGs are increased by 0.0028 and 0.0219 (2\(\pi/c\)), respectively. Thus, the first two PBGs can be tuned by the filling factor of anisotropic dielectric spheres. This can be explained in physics that increasing anisotropic dielectric filling factor means the space averaged dielectric constant of such 3D MPPCs becoming larger [21, 22]. Fig.2 also reveals that the relative bandwidth of both PBGs increase first then decrease with increasing \(f\). The maximum relative bandwidths of first two PBGs are 0.0703 and 0.0324, which can be found at the cases of \(f=0.3\) and 0.1, respectively. As mentioned above, the filling factor of anisotropic dielectric spheres is an important parameter which need be chosen. It also is noticed that if anisotropic dielectric filling factor is small enough and close to null, the 3D MPPCs can be looked as a magnetized plasma block. The flatbands regions will disappear.

IV. Conclusion

In summary, the band Faraday effects in the 3D dispersive PCs composed of the homogeneous anisotropic dielectric spheres (the uniaxial material) immersed in the magnetized plasma background with fcc lattices are theoretically investigated by the plane wave expansion (PWE) method, as the Faraday effects of magnetized plasma are considered. The equations for calculating the anisotropic PBGs in the first irreducible Brillouin zone are theoretically deduced. Based on the calculated results, some conclusion
can be drawn. Compared with the same structure composed by the isotropic dielectric spheres in the air, the 3D MPPCs doped by the uniaxial material can obtain complete PBGs and one flatbands region also can be achieved. The flatbands caused by the existence of surface plasmon modes which stem from the coupling effects between the magnetized plasma. The relative bandwidths of first two PBGs will increase first then decrease as the anisotropic dielectric filling factor is increased in a certain range. It also is noticed that if plasma filling factor is large enough and close to one, the 3D MPPCs can be seen as a magnetized plasma block. The flatbands region will disappear. The 1st PBG has a larger relative bandwidth compared to 2nd PBG. As mentioned above, we can acquire the PBGs in expected frequency and take advantage of the uniaxial material to obtain the complete PBGs for 3D MPPCs with fcc lattices by selecting the appropriate parameters as the Faraday effects are considered.

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