Quantum State Propagation in Quantum Wireless Multi-hop Network based on EPR pairs

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Abstract—Quantum wireless multi-hop network (QWMN) is one emerging area which is relative to quantum communication and information. Long distance delivery of quantum information in network is an important problem in future. This results in our study on quantum state propagation in QWMN by using EPR pairs. In this paper, we present and prove method to transfer quantum state in QWMN. With this method, quantum state can be teleported from source to destination and intermediate nodes can complete teleportation in parallelism and independently in QWMN. The time complexity of this method is independent of number of hops. In addition, we verify that EPR-pair bridging is still applicable for transmitting quantum state in n-hop routing and extend the logical relation of result to n-hop case. Based on these studies, we can propagate quantum states instantaneously in QWMN and destination node can recover the state only with the measurement outcomes of intermediate nodes.

Keywords: Quantum state propagation, teleportation, quantum wireless multi-hop network, EPR pairs.

I. INTRODUCTION

Quantum communication network is the combination of quantum and information theory. The advanced communications and quantum networking technologies offered by quantum information processing will revolutionize the traditional communication and networking methods [1]. Due to its potential impact and significance, quantum communication has drawn more and more attentions in the decade.

The fascinating way of propagation in quantum communication is teleportation, which is also the key to quantum network. Quantum teleportation, proposed by Bennett [2], employing a particular property of quantum called quantum entanglement, allows quantum devices to transmit a quantum state to another node at a distance. Shared entanglement is essential to achieve quantum teleportation, which can be accomplished by generating an EPR pair and distributing the pair to the source and destination in advance [3].

An important problem of quantum communications in future is the long distance delivery of quantum information. The fidelity of the entanglement decreases during the transmission through the noisy quantum channel. Considering the quantum entanglement state can be generated and distributed between smaller segments, quantum repeaters are used as a router to sharing and distributing quantum entanglement [4]. However, the repeaters are connected by optical fibers while they are static. In the field of wireless network, where nodes usually are mobile, the EPR pairs cannot be distributed to wireless quantum device through the air. As a result, wireless quantum devices cannot set up EPR pairs instantaneously. In the meantime, it is unreasonable for a limited storage mobile device to keep many EPR pairs entangled with all possible communication parties for the future use in wireless environment. So, new quantum mechanisms are established to realize the teleportation even when the two sites do not share EPR pairs mutually. Cheng et al. [5] introduced the first quantum routing mechanism in wireless communication network using quantum relay and EPR-pair bridging, and analyzed the process of the propagation in two hops after setting up the routing path. Yu et al. [6] presented one on-demand quantum routing protocol in wireless ad-hoc network, employing the both approximation method to set up the quantum channel in three hops. Quantum communication in distributed wireless sensor network is discussed in [7], but it mainly solved the quantum key distribution (QKD) and quantum cryptography problem in WSN.

The aim of this paper is to solve the information propagation problem in quantum wireless multi-hop network. By analyzing and deducing, this paper extends the EPR-pair bridging to n-hop situation and proves that teleportation mechanism is also applicable in wireless multi-hop quantum networks. In the meanwhile, the paper analyzes the process of teleportation and demonstrates the XOR relation between measured outcomes of intermediate nodes.

The rest of this paper is organized as follows. In section II, the model of quantum wireless multi-hop network is given. Teleportation in quantum wireless multi-hop network is discussed in section III. More general cases of teleportation are deduced in section VI. Finally, in section V, conclusions are drawn.

II. QUANTUM WIRELESS MULTI-HOP NETWORK MODEL

Quantum wireless multi-hop network, as is shown in Fig.1, is composed of wireless quantum nodes (WQN) with both quantum and wireless communication capabilities. There are
two kinds of communication channels between WQNs: classical channel for transmitting classical radio data with traditional radio transceivers and quantum channel for teleporting quantum state from a source node to a faraway destination node. A quantum bit (qubit) is a unit of quantum information, two possible states for a qubit are \( |0\rangle \) and \( |1\rangle \) so that one qubit can be symbolized in the linear combinations form of \( |\phi\rangle = \alpha |0\rangle + \beta |1\rangle \), called superposition, where \( \alpha \) and \( \beta \) are complex numbers. We cannot examine a qubit to determine its quantum states, only can get the result 0 with probability \( |\alpha|^2 \) or the result 1 with probability \( |\beta|^2 \). Naturally, \( |\alpha|^2 + |\beta|^2 = 1 \), since the probabilities must sum one. Every state in a superposition would be transformed simultaneously.

In quantum networks, the quantum state is propagated by the means of quantum teleportation, applying the property of EPR pairs. An EPR pair is a two-qubit system in maximally entangled state and can be denoted by \( |1\sqrt{2} (|00\rangle + |11\rangle) \). The quantum communication channel exits between arbitrary two nodes, which share at least one couple of entangled EPR pairs. Each node owns one qubit of the EPR pair. With the aid of the EPR pair, by only performing local operations and classical communication, these two nodes can propagate quantum states with each other.

In quantum wireless multi-hop network, routing mechanism is needed to choose one routing path from source to destination before quantum teleportation starts. This routing includes one classical communication path and one quantum communication path. Here, we discuss quantum state propagation rather than the routing, so we assume that the routing path has been selected appropriately to form one line of multi-hop nodes.

III. TELEPORTATION THROUGH BRIDGING

In quantum wireless multi-hop network, it is unrealistic and impossible to share EPR pairs with all the other potential destination nodes in network. To achieve quantum teleportation even if the source and destination do not share any EPR pair mutually, quantum relay and EPR-pair bridging are proposed.

The principle of quantum relay is to perform the quantum teleportation hop by hop from the source to destination when there are intermediate nodes equipped with EPR-pair shared with upstream and downstream nodes in between. In other words, every intermediate node on the routing path would recover the state of the quantum teleported by the upstream nodes, then teleport it again to the downstream nodes until it reaches the destination. The quantum relay indeed teleport the quantum state in multi-hop network, but the target qubit is transmitted through each intermediate node, this is lack of security and privacy. In the meantime, the relay must be performed hop by hop so that the complexity depends linearly on the number of the intermediate nodes.

The EPR-pair bridging speeds up the whole quantum relay process by performing the quantum teleportation at each intermediate nod in parallelism. The intermediate nodes are able to establish the entanglement-assisted quantum channel between its upstream and downstream nodes.

![Fig.2 Quantum circuit for EPR-pair bridging](image)

The entire quantum circuit for EPR-pair bridging as mention in [5] is given in Fig.2. The circuit is composed with quantum utility gates and measuring tool. The quantum gate is the similar to the logic gate in classical computer circuit to transform quantum state. \( X, Z, \text{Hadamard} \) and CNOT gates are four important quantum gates used in quantum circuit. The quantum \( X \) gate can be used to flip its state \( |0\rangle \) and \( |1\rangle \). The \( Z \) gate is used to flip the phase of the state \( |1\rangle \). In other words, it can exchange \( \alpha |0\rangle + \beta |1\rangle \) and \( \alpha |0\rangle - \beta |1\rangle \). The Hadamard gate is one of the most useful quantum gates, which can turn \( |0\rangle \) into \( 1/\sqrt{2} (|0\rangle + |1\rangle) \) and turn \( |1\rangle \) into \( 1/\sqrt{2} (|0\rangle - |1\rangle) \). The Controlled-NOT (CNOT) gate is used to process two quits. When the first qubit is in state \( |0\rangle \), it does nothing. Conversely, when the first qubit is in state \( |1\rangle \), then it would flip the second qubit state. In other words, the CNOT gate is used to exchange \( |01\rangle \) and \( |11\rangle \).

In Fig. 2, single lines represent quantum data and the double lines stand for classical measurement results. Without the dotted portion, \( Alice \) is the source and \( Bob \) is the destination, while \( Candy \) is intermediate node between \( Alice \) and \( Bob \). It can be seen from Fig.2 that \( Alice \)'s second qubit and \( Candy \)'s first qubit shared the EPR-pair as \( 1/\sqrt{2} (|00\rangle + |11\rangle)\) and EPR-pair \( 1/\sqrt{2} (|00\rangle + |11\rangle)\) is shared between \( Candy \)'s second qubit and \( Bob \). The suffixes (e.g. \( A2, C1 \), etc.) stand for the ordinal number at the owner’s side. For example, the state with the \( A2 \) represents \( Alice \)'s second qubit. When \( Alice \) intends to deliver a qubit \( |\psi\rangle = \alpha |0\rangle + \beta |1\rangle \) to \( Bob \), \( Alice \) and \( Candy \) send their second quits through a CNOT gate. After that they perform \( H \) gate on their first quits, then they measure the quits and transmit the measurement outcomes to \( Bob \) through classical channel. Once \( Bob \) receives the two classical bits, he knows that \( Alice \) has teleported a qubit state to him. According to the two classical bits received, the quantum state \( |\psi\rangle \) can be reconstructed by appropriate operation.

In the beginning, the five-qubit system can be represented as:

\[
\frac{1}{2\sqrt{2}} (|00\rangle_{AC1} + |11\rangle_{AC1}) \otimes (|00\rangle_{AC1} + |11\rangle_{AC1})
\]

(1)

After \( Alice \) and \( Candy \) both send their second quits through a CNOT gate and their first quits through a \( H \) gate, the entire system would turn into:
\[
\begin{align*}
&\frac{1}{4}\left[ (|0\rangle_{A1C}+|1\rangle_{A1C})\otimes(|0\rangle_{A2C}+|1\rangle_{A2C})\otimes(\alpha(0)+\beta(1))_{B1} \\
&\quad+\frac{1}{2}\sqrt{2}\left[ |0\rangle_{A1C}+|1\rangle_{A1C}\right]\left[|0\rangle_{A2C}+|1\rangle_{A2C}\right]\otimes(\alpha(0)+\beta(1))_{B1} \\
&\quad+\frac{1}{2}\sqrt{2}\left[ |0\rangle_{A1C}+|1\rangle_{A1C}\right]\left[|0\rangle_{A2C}+|1\rangle_{A2C}\right]\otimes(\alpha(0)-\beta(1))_{B1} \\
&\quad+\frac{1}{2}\sqrt{2}\left[ |0\rangle_{A1C}+|1\rangle_{A1C}\right]\left[|0\rangle_{A2C}+|1\rangle_{A2C}\right]\otimes(\alpha(1)-\beta(0))_{B1} \right] \\
\end{align*}
\] (2)

Considering the logical relation and utilizing the digital logic method, the expression can also be rewritten as:

\[
\begin{align*}
&\frac{1}{4}\left[ (A1\oplus C1\otimes \bar{T}2\otimes C2\otimes(\alpha(0)+\beta(1))_{B1} \\
&+A1\oplus C1\otimes (A2\otimes C2\otimes(\alpha(0)+\beta(0))_{B1} \\
&+(A1\oplus C1\otimes (A2\otimes C2\otimes(\alpha(0)-\beta(1))_{B1} \\
&+(A1\oplus C1\otimes (A2\otimes C2\otimes(\alpha(1)-\beta(0))_{B1} \right] \\
\end{align*}
\] (3)

From expression (3), we can see that Bob would have the information of his qubit state once he is informed of Alice’s and Candy’s two qubit states. After receiving the measurement outcomes, Bob can fix up his own qubit to recover by either applying nothing, X gate, Z gate, or both X and Z gate. We can depict the relation as following: if 1) the result of Alice’s first qubit XOR Candy’s first qubit is 0 and 2) the result of Alice’s second qubit XOR Candy’s qubit is 1, then Bob has to fix up his qubit by applying quantum X gate. Likewise, if 1) the result of Alice’s first qubit XOR Candy’s qubit is 1 and 2) the result of Alice’s second qubit XOR Candy’s qubit is 0, then Bob can fix up his state by applying quantum Z gate. The basic quantum circuit just takes 2-hop case into consideration; we will verify its validity in more general cases.

IV. TELEPORTATION IN QUANTUM WIRELESS MULTI-HOP NETWORK

In the long distance wireless transmission, the fidelity of the entanglement decreases as the distance increases and the propagation radius of classical wireless communication is limited, it is more possible to teleport quantum state in multi-hop way from source to destination with more than one intermediate. Therefore, it is necessary to extend the EPR-pair bridging to n-hop network and verify the validity and feasibility.

A. 3-hop case

We start the work from two intermediate nodes, adding intermediate node David (i.e dotted portion) on the routing path. David also shares EPR-pair with Candy and Bob. So the system can be represented as follow:

\[
\frac{1}{2}\sqrt{2}|\psi\rangle\otimes(|0\rangle_{A1C}+|1\rangle_{A1C})\otimes(|0\rangle_{A2C}+|1\rangle_{A2C})\otimes(|0\rangle_{D2C}+|1\rangle_{D2C})
\] (4)

Because the system can perform asynchronously, source and intermediate nodes do not need to coordinate before they perform each gate. So After Alice, Candy and David all send their second qubits through CNOT gates and their first qubits through H gates, the entire system state can be seen as the result of system state of 2-hops direct product result of David’s qubits going through CNOT and H gate respectively. The expression would change into:

\[
\begin{align*}
&\frac{1}{4}\sqrt{2}\left[ a \otimes B \left[ (|0\rangle+\beta|1\rangle_{D1}\otimes(|0\rangle_{D2}+|1\rangle_{D2})\right] \\
&+b \otimes B \left[ (|0\rangle-\beta|1\rangle_{D1}\otimes(|0\rangle_{D2}+|1\rangle_{D2})\right] \\
&+c \otimes B \left[ (|1\rangle+\beta|0\rangle_{D1}\otimes(|0\rangle_{D2}+|1\rangle_{D2})\right] \\
&+d \otimes B \left[ (|1\rangle-\beta|0\rangle_{D1}\otimes(|0\rangle_{D2}+|1\rangle_{D2})\right] \right] \\
\end{align*}
\] (5)

In expression (5), factors \(a, b, c, d\) represent the four possible terms entangled qubits. For example, \(a\) replaces the possible term \((|0\rangle_{A1C}+|1\rangle_{A1C})\otimes(|0\rangle_{A2C}+|1\rangle_{A2C})\). Also, operator B is introduced here to denote \((H\otimes I)\) CNOT operation that the first qubit goes through CNOT gate and second qubit H gate. Expression (5) can be expanded into:

\[
\begin{align*}
&\frac{1}{8}\left[ a \otimes |0\rangle_{D1D2}+b \otimes |0\rangle_{D1D2} \\
&+c \otimes |1\rangle_{D1D2}+d \otimes |1\rangle_{D1D2} \right] \otimes (\alpha(0)+\beta(1))_{B1} \\
&+\frac{1}{8}\left[ a \otimes |0\rangle_{D1D2}+b \otimes |0\rangle_{D1D2} \\
&+c \otimes |0\rangle_{D1D2}+d \otimes |1\rangle_{D1D2} \right] \otimes (\alpha(1)+\beta(0))_{B1} \\
&+\frac{1}{8}\left[ a \otimes |0\rangle_{D1D2}+b \otimes |1\rangle_{D1D2} \\
&+c \otimes |0\rangle_{D1D2}+d \otimes |0\rangle_{D1D2} \right] \otimes (\alpha(0)-\beta(1))_{B1} \\
&+\frac{1}{8}\left[ a \otimes |1\rangle_{D1D2}+b \otimes |1\rangle_{D1D2} \\
&+c \otimes |1\rangle_{D1D2}+d \otimes |0\rangle_{D1D2} \right] \otimes (\alpha(1)-\beta(0))_{B1} \right] \\
\end{align*}
\] (6)

The expression (6) can be rewritten and then we can get the final formula expression:

\[
\begin{align*}
&\frac{1}{8}\left[ A1\oplus C1\otimes D1\otimes A2\otimes C2\otimes D2\otimes(\alpha(0)+\beta(1))_{B1} \\
&+(A1\oplus C1\otimes D1\otimes A2\otimes C2\otimes D2\otimes(\alpha(0)+\beta(0))_{B1} \\
&+(A1\oplus C1\otimes D1\otimes A2\otimes C2\otimes D2\otimes(\alpha(0)-\beta(1))_{B1} \\
&+(A1\oplus C1\otimes D1\otimes A2\otimes C2\otimes D2\otimes(\alpha(1)-\beta(0))_{B1} \right] \\
\end{align*}
\] (7)

Similarly, when Bob got the measurement outcomes of intermediate nodes including David’s, he can apply specific quantum gate to recover \(|\psi\rangle\) and we can get the similar relation to 2-hop situation.

B. More general cases

According to the result of the 2-hop and 3-hop cases, we consider system expression at n-1 hop situation as following:

\[
\begin{align*}
&\frac{1}{2^{n+1}}\left[ N_{2,1}\otimes N_{2,2}\otimes\cdots\otimes N_{2,n}\otimes(\alpha(0)+\beta(0))_{B1} \\
&+N_{3,1}\otimes N_{3,2}\otimes\cdots\otimes N_{3,n}\otimes(\alpha(1)+\beta(0))_{B1} \\
&+(N_{4,1}\otimes N_{4,2}\otimes N_{4,3}\otimes\cdots\otimes N_{4,n}\otimes(\alpha(0)-\beta(1))_{B1} \\
&+(N_{4,1}\otimes N_{4,2}\otimes N_{4,3}\otimes\cdots\otimes N_{4,n}\otimes(\alpha(1)-\beta(0))_{B1} \right] \\
\end{align*}
\] (8)

There are \(n\) nodes on the n-1 hops route. In expression (8), \(N\) represents the qubit measurement outcome of the nodes on the route while the subscript denotes the number of the nodes and the number of the qubit. So, \(N_{2,1}\) denotes the measurement outcome of the second node’s first qubit. Similarly, \(N_{n,1,2}\) denotes the outcome of the (n-1)th node’s second qubit.
When we add one intermediate node, the route is n-hop long, as shown in Fig.3. We just need to use the result of n-1-hop case to direct the measurement outcome of the new node after going through CNOT and H gate, so that the entire system state turn into:

\[
\frac{1}{2^{n-1}} \sum_{\alpha} \left( \left| 00 \right>_{n-1} \otimes \left| 1 \right>_{n} \right) \left( N_{1,1} \otimes \left( N_{1,2} \otimes \cdots \left( N_{n-1,1} \otimes B(\alpha(0) + \beta(1))_{n+1} \right) \right) + N_{1,2} \otimes \left( N_{1,1} \otimes B(\alpha(0) + \beta(1))_{n+1} \right) + \cdots + N_{n-1,1} \otimes \left( N_{1,2} \otimes \cdots \left( N_{1,1} \otimes B(\alpha(0) + \beta(1))_{n+1} \right) \right) \right),
\]

(9)

We can rewrite the expression (9) according to the way of expression (3) and (8):

\[
\frac{1}{2^n} \sum_{\alpha} \left( \left| 00 \right>_{n-1} \otimes \left| 1 \right>_{n} \right) \left( N_{1,1} \otimes \left( N_{1,2} \otimes \cdots \left( N_{n-1,1} \otimes B(\alpha(0) + \beta(1))_{n+1} \right) \right) + N_{1,2} \otimes \left( N_{1,1} \otimes B(\alpha(0) + \beta(1))_{n+1} \right) + \cdots + N_{n-1,1} \otimes \left( N_{1,2} \otimes \cdots \left( N_{1,1} \otimes B(\alpha(0) + \beta(1))_{n+1} \right) \right) \right) + \sum_{\beta} \left( \left| 00 \right>_{n-1} \otimes \left| 1 \right>_{n} \right) \left( N_{1,1} \otimes \left( N_{1,2} \otimes \cdots \left( N_{n-1,1} \otimes B(\alpha(0) + \beta(1))_{n+1} \right) \right) + N_{1,2} \otimes \left( N_{1,1} \otimes B(\alpha(0) + \beta(1))_{n+1} \right) + \cdots + N_{n-1,1} \otimes \left( N_{1,2} \otimes \cdots \left( N_{1,1} \otimes B(\alpha(0) + \beta(1))_{n+1} \right) \right) \right) + \sum_{\alpha, \beta} \left( \left| 00 \right>_{n-1} \otimes \left| 1 \right>_{n} \right) \left( N_{1,1} \otimes \left( N_{1,2} \otimes \cdots \left( N_{n-1,1} \otimes B(\alpha(0) + \beta(1))_{n+1} \right) \right) + N_{1,2} \otimes \left( N_{1,1} \otimes B(\alpha(0) + \beta(1))_{n+1} \right) + \cdots + N_{n-1,1} \otimes \left( N_{1,2} \otimes \cdots \left( N_{1,1} \otimes B(\alpha(0) + \beta(1))_{n+1} \right) \right) \right)
\]

(10)

Note that the expression (10) has the same form with expression (8), so we can conclude that adding one intermediate node on the route has no effect on the logical relation. For fixing up its own qubit to recover the state \(|\psi\rangle\), the destination node only needs to know the measurement outcomes of intermediate nodes on the routing path. These outcomes were transmitted in classical channel by each intermediate node respectively and independently, not relying on each other.

### V. CONCLUSION

We present study on the quantum state propagation in quantum wireless multi-hop network. Our study shows that quantum states can be transmitted from source node to destination node based on EPR pairs. We verify that EPR-pair bridging is still applicable in n-hop case even if they do not share EPR pairs mutually. Through quantum circuit of n-hop network, intermediate nodes can make measurement independently and the quantum state can be transferred successfully. Therefore, the time complexity of teleporting a quantum state is independent of the number of routing hops. In addition, we also demonstrate that to recover the transmitted quantum state, the destination node just need to know measurement outcomes of all intermediate nodes on the routing path and fix up its own qubit to corresponding operation and XOR relation of measurement outcomes and operations is illustrated as well.

### REFERENCES


