Abstract: Superconducting hot-electron-bolometer (HEB) mixer is the most sensitive detector in the terahertz (THz) frequencies. The design of RF coupling circuit is very crucial to the mixer’s sensitivity. In this paper, we present a theoretical model for the calculation of the feed point matching impedance of the coupling circuit, and demonstrate the simulation results with the aid of 3D electromagnetic simulation software (HFSS) and a lumped-gap source method similar to quasi optical antennas in the frequency range of 0.9-1.3 THz. The influences of the slice tolerance of substrate thickness, the high-order modes, and the feed-point size on the embedding impedance at 0.9-1.3 THz have been studied in detail. The mixer’s embedding impedance is simulated to be around 35-j10 in the whole working band whose bandwidth can reach 36%. The simulation results are in good agreement with the theoretical results, which provide helpful instructions for the future developments of ultra-wide band and highly sensitive superconducting hot-electron-bolometer (HEB) mixer.

Key words : superconducting hot-electron-bolometer (HEB) mixer; embedding impedance; wide band matching; high-order mode; substrate thickness

I. INTRODUCTION

Superconducting hot-electron-bolometer (HEB) mixer is the most sensitive detector in the terahertz frequency band, based on the fixed-tuned waveguide coupling circuit and the good performance of the beam. It has been widely used in radio telescopes on the ground and space[1-3].

Embedding impedance is an important parameter to characterize the coupling efficiency between the mixer and the coupling circuit, which furthermore affects the mixer’s sensitivity[4,5]. In order to achieve the high sensitivity and wide frequency bandwidth suitable for astronomical observation in terahertz frequency band, a fixed-tuned waveguide coupling circuit with ultra-wide band is proposed from 0.9 to 1.3 THz. We present the equivalent circuit model of the RF coupling circuit according to transmission line theory. The model is demonstrated to be in good agreement with the calculated results by using HFSS software. In addition, the effects of substrate thickness, the low-pass microstrip filter, and the feedpoint size on the embedding impedance have been investigated. This paper aims to optimize the structure of the transition of waveguide to microstrip and low-pass microstrip filter, further improves the embedding impedance matching to the mixer and increases the working frequency bandwidth of the mixer.

The mixer coupling circuit working at 0.9-1.3 THz, which presented in this paper will be used in Atacama Submillimeter Telescope Experiment Telescope in Japan, to achieve the observation of astrochemistry molecular spectral line.

II. THEORETICAL CALCULATION

Figure 1 shows the schematic of the waveguide coupling circuit of the superconducting HEB mixer. RF signal input into the waveguide port, then pass through the transition of waveguide to microstrip, finally enter the HEB mixer in the feed point. The two symmetric low-pass microstrip filters (i.e., choke filter) are used to choke the RF signal and transmit the low frequency signal after mixing.

The feed point located at the superconducting HEB mixer (of 50-Ω normal-state resistance), which has a 2-μm width and 0.2-μm height, as shown in Fig. 1 (a). The matching impedance at lumped-gap source port is made up of several parts: the main mode transfer impedance $Z_{g_{10}}$, backshort impedance under the back part of short circuit $Z_{bs}$, symmetrical impedance $Z_{chk1}$ and $Z_{chk2}$ of low-pass microstrip filter, as shown in Fig. 1 (b). The transition

![Fig.1 (a) Front view and (b) side view of a waveguide HEB mixer](image-url)
probe from waveguide to microstrip (shadows in Fig. 2) are symmetrical according to the location of feed point.

The high-order modes probably exist at the feed point due to the discontinuity of the transition of waveguide to microstrip. These non-propagating mode changes the E-field distribution of the feed point. We present the equivalent circuit model of the propagation of the high-order modes, as shown in Fig. 3.

The impedance of the waveguide main mode TE_{10} and high-order modes are given by equation (1) and (2).

\[ Z_{g10} = \frac{b}{a} \sqrt{\frac{\mu_0}{\varepsilon_0}} = 33 \Omega \]

\[ Z_{mn} = i\eta \frac{b}{a} \frac{c}{2 \pi f} \left( \frac{2 \pi f}{c} \right)^2 - \frac{m \pi}{b} \right) (2 - \delta_n) \gamma_{mn}^{-1} \]

\[ \delta_n = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases} \]

\[ \gamma_{mn} = \alpha + i \beta_{mn} \]

\[ \alpha = \frac{8.686}{\eta \sigma} \frac{b}{a} \frac{c}{2 \pi f} \left[ 1 - \left( \frac{f}{f_c} \right)^2 + 1 \right] \]

\[ \beta = \sqrt{\frac{2 \pi f}{c}} - \frac{m \pi}{b} - \frac{n \pi}{b} \]

In the equations, \( c \) is the speed of light in a vacuum, under the temperature of 300 K, \( \delta \) equal to 42.5 * 10^6 (\( \Omega \))^{-1}, about 65 nm, \( Z_{mn} \) is the impedance of the TE_{mn} and TM_{mn} modes.

The position of the metal and geometry dimension in waveguide is shown in Fig. 2, the coupling coefficients are shown in equation (6, 7).

\[ k_{pm} = \sin(m \pi \frac{s}{a}) \sin(m \pi \frac{w}{2a})(m \pi \frac{w}{2a})^2 \]

\[ k_{gm} = \cos(n \pi \frac{h}{b}) \sin(n \pi \frac{g}{2b})(n \pi \frac{g}{2b})^2 \]

Waveguide back is a short circuit, the resistance under the main mode is a function of frequency, as shown in equation (8).

\[ Z_{bs} = Z_{g10} \left( \frac{Z + Z_{g10} \tanh(\gamma_{10} l_{bs})}{Z + Z_{g10} \tanh(\gamma_{10} l_{bs})} \right) \]

In superconducting HEB mixer, due to the symmetrical structure of two low pass filter, \( Z_{lbd} \) and \( Z_{chd} \) are equivalent. Considering each part’s impedance contributions, the feeding point impedance can be calculated by equivalent circuit of Fig. 4.

Concerning of the capacitance effect of medium plate substrate, a capacitance constant \( Y_{esf} = 12.3 \ fF(b-7) \) should be added into the circuit. In Fig. 4, \( X_L \) and \( Y_R \) represent the parasitic impedance of high-order modes, as shown in the following expressions.

\[ X_L = \sum_{n=2}^{2p} k_{pm}^2 Z_{mn} \]

\[ Y_R = \sum_{m=1}^{2} \sum_{n=1}^{2} Z_{mn} (k_{pm}^2)^{-1} \]

For symmetric structure of probe \((s = a / 2, h = b / 2)\), obtained from equation (6, 7), there is no coupling coefficients. In frequency range of 0.9-1.3 THz, the influence of high-order modes are very small, thus \( X_L \) and \( Y_R \) approximate to zero.

### III. SCALE-MODEL OPTIMIZATION

The transmission of multiple modes will lead to dispersion phenomena and signal distortions, and reduce the transmission power into the HEB mixer. To ensure that only dominant mode of waveguide propagate in the range of 0.9-1.3 THz, the wavelength must satisfy the following conditions:

\[ \lambda_{c01} < \lambda < \lambda_{c10} \]

\[ \lambda_{c20} < \lambda < \lambda_{c10} \]

For single-mode propagation, the most suitable size ranges below:

\[ a \lessapprox \lambda \lessapprox 2a \text{ and } \frac{b}{a} \lessapprox 2 \]

In conclusion, within the scope of 0.9-1.3 THz, the width and height of the waveguide is chose to be 200 \( \mu \)m and 100 \( \mu \)m.
respectively. Fig. 5 is the model used in the simulation of the HFSS, colored areas are microstrip lines, to choke RF signals and transmit the low frequency signal.

Microstrip low-pass filter\[^{[8]}\] in mixer is designed on the single crystal substrates ($\varepsilon = 4.65$), placed in a rectangular cavity with the size of 73 $\mu$m by 73 $\mu$m. The upper and lower microstrip filter is symmetrically arranged. High-impedance line works similar to series inductance, low-impedance line similar to parallel capacitor. Characteristic impedance of the filter ranges as shown follows:

$$Z_{OC} < Z_a < Z_{QL} \quad (14)$$

$Z_{OC}$, $Z_{QL}$ corresponding to high- and low-impedance line characteristic impedances. Low-impedance line works as short circuit. The high-impedance line is equivalent to open circuit, and the width is limited for fabrication process and influence line of current-carrying capacity. In order to facilitate the optimization of the model, one part of the microstrip filters will be investigated for simulation, as shown in Fig. 6.

Initial size of the microstrip line is showed in Tab. I. We optimize the width and the length of impedance lines such as $a_1$, $a_2$, $a_3$, $b_1$, $b_2$, $b_3$, as shown in Fig. 6.

As seen from the reflection coefficient, with the increase of frequency, low pass filter has a bad performance at the high frequency band. With the increase of length of $b_1$, the reflection coefficient at higher frequency approach to 1. It means the RF signal is choked from IF port and enter the mixer, thus the size of the $b_1$ is fixed 70 $\mu$m, as shown in Fig. 7. Considering the entire optimizations, the simulation reflection coefficient is shown in Fig. 8.

IV. OPTIMIZATION OF SUBSTRATE THICKNESS

We calculated the embedding impedance for different substrate thicknesses of the HEB mixer. The embedding impedance is calculated by the equation of $Z_{emb} = Z_0 \cdot (1 + \Gamma)/(1 - \Gamma)$, here $\Gamma$ is complex reflection coefficient at feed point. As exhibited in Fig. 9, the embedding impedance change is most flat in the whole frequency band when the substrate thickness is 35 $\mu$m. We consider the effect of higher-order mode in the HFSS simulation, and set the feed port to be multi-mode. The simulation results indicate that the higher-order modes have slight impact on the embedding impedance. The simulated embedding impedance $Z_{emb}$ is approximately 35-j10 $\Omega$, which is close to the impedance of the mixer(35 $\Omega$), it can achieve broad-band impedance matching within the scope of 0.9-1.3 THz.

We compare the calculation of the equivalent circuit theory model in Fig. 4 with the simulation results by using HFSS, and found that the two methods show basically identical result. The tolerance of the resistance and reactance is within 5 $\Omega$, as shown in Fig. 10.
Fig. 9 Embedding impedance at feed point when the substrate thickness is 33μm, 35μm, 37μm.

Fig. 10 Comparison of the embedding impedance between the theoretical calculation and HFSS simulation.

V. CONCLUSION

We have successfully designed a broadband terahertz coupling circuit of superconducting HEB mixer. We have optimized the coupling structure of the HEB mixer such as the dimensions of the rectangular waveguide, the microstrip filter, and the substrate thickness. The simulation result shows that the embedding impedance of the mixer is about 35-j10 in the frequency range of 0.9-1.3 THz. We present the equivalent coupling circuit with taking account into high-order modes. The theoretical calculation results demonstrate the simulation results by HFSS with the error below 5 Ω. The relative working bandwidth reaches 36%. This paper is of great significance to implement the ultra-wide band terahertz mixer in the future.

REFERENCES